

Contrast transfer function of the light diffusion in photographic emulsions

By L. O. HENDEBERG

With 30 figures in the text

ABSTRACT

Due to the method employed here, the photographic emulsion preserves the sinusoidal shape and linearity of a sinusoidal line pattern. This enables one to determine experimentally the Fourier transform $|F_{\text{em}}(N)|$ of the light distribution function produced by an emulsion. A sinusoidal line pattern with several previously chosen space frequencies is photographed on an emulsion, whereby the transform for the combination of optical system and emulsion is obtained. The influence of the optics is eliminated numerically by means of an experimental knowledge of its Fourier transform $|F_{\text{opt}}(N)|$. The function $|F_{\text{em}}(N)|$, however, is dependent on the emulsion-developer combination, the spectral range used in the exposure of the emulsion, the storage of the film, the particular batch of the emulsion which is investigated, etc. It is therefore difficult in general to fit a one-parameter function to the light diffusion distributions. The assumptions of Frieser and Sayanagi fit best for certain combinations while functions of the type $[1 + cx^2]^{-1}$ are best for others. These inverses of the experimental Fourier transforms $|F_{\text{em}}(N)|$ are obtained by means of an electronic calculating machine.

Introduction

This work reports an experimental investigation of light distribution function of several photographic emulsions. It is based on the multiplication of contrast transfer function [2]. The influence of the optical system was eliminated by means of numerical calculations. The original experimental arrangement, on which this work is based, has been reported by Ingelstam, Djurle and Sjögren [7]. In this connection, the basic works of Gaultier du Marache [5] and Frieser [4] should also be mentioned.

The one-dimensional intensity distribution of a luminous object is characterized by the space function $f_o(x)$. This function can be expressed as a summation of terms with an increasing space frequency N (N = number of sinusoidal periods per mm):

$$f_o(x) = \int_{-\infty}^{+\infty} C_o(N) \cdot e^{i2\pi Nx} dN \quad (1)$$

The light distribution function $f_i(x)$, which characterizes the distribution of the image intensity, can be written in a similar way:

$$f_i(x) = \int_{-\infty}^{+\infty} C_i(N) \cdot e^{i2\pi Nx} dN \quad (2)$$

The contrast transfer function (written hereafter as CT) of the image transmitting system, e.g. $|F_{\text{opt}}(N)|$ for a lens system, is given by the ratio between the contrast of the sinusoidal waves for the image and that for the object:

$$|F_{\text{opt}}(N)| = \frac{C_i(N)}{C_o(N)} \quad (3)$$

The contrast of a sinusoidal intensity distribution is defined as the ratio between the amplitude and the mean intensity:

$$C(N) = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \quad (4)$$

In principle, the light distribution function $f(x)$ describes the intensity just as completely as the CT function or the Fourier transform $F(N)$. But, from a practical point of view, the CT function is to be preferred. Thus, using CT , it is easier to calculate the image when the transmitting system consists of a cascade of n linearly working elements situated one after the other, e.g. an optical system together with a photographic emulsion. The $F_i(N)$ of the image is obtained by a multiplication point by point of the CT functions $F_m(N)$ of the element:

$$F_i(N) = \prod_{m=1}^n F_m(N) \quad (5)$$

If, for example, an object intensity is reproduced by means of an optical system, then the image intensity can be characterized by its CT function in the following way:

$$F_i(N) = F_o(N) \cdot F_{\text{opt}}(N) \quad (6)$$

The corresponding procedure is more complicated when the object intensity $f_o(x)$ and the light diffusion function $\phi_{\text{opt}}(x)$ of the optical system are known. The simple multiplication calculation is then replaced by a convolution

$$f_i(x') = \int_{-\infty}^{+\infty} f_o(x) \cdot \phi_{\text{opt}}(x - x') dx \quad (7)$$

This formula can be used when the reproduction is made by several transfer elements situated in series. But the mathematical calculations become more complicated the greater the number of elements.

This investigation of the CT function $F_{\text{em}}(N)$ of the photographic emulsion was based on the formula given above. It is difficult experimentally to measure directly the light diffusion function $\phi_{\text{em}}(x)$. The transform $F_{\text{em}}(N)$ is determined instead. The light path should be the same as that when using the photographic emulsion. The emulsion was therefore illuminated in these experiments by means of a cone of light from a photographic objective.

In this investigation, the combined CT of the system, viz. $|F_{\text{tot}}(N)|$, was measured. In a complimentary investigation the alteration in object intensity, when only the optical system was used, was measured. The function obtained by point division of the two CT functions

$$|F_{\text{em}}(N)| = \frac{|F_{\text{tot}}(N)|}{|F_{\text{opt}}(N)|} \quad (8)$$

is what is wanted. By means of an inversion of the Fourier transform $F_{\text{em}}(N)$, the light distribution function $\phi_{\text{em}}(x)$ of the emulsion can be calculated in the following way:

$$\phi_{\text{em}}(x) = \int_{-\infty}^{+\infty} F_{\text{em}}(N) e^{+i2\pi Nx} dN \quad (9)$$

Such a calculation can be carried out by means of an electronic calculating machine.

The object consisted of a slit. The distribution of its intensity is denoted by the functions $f_s(x)$ or $F_s(N)$. An optical system with a light distribution function $\phi_{\text{opt}}(x)$ reproduces, in the image plane, an image intensity i.e. an aerial image $f_{\text{air}}(x)$. It is found that the intensity distribution $f_s(x)$ has been changed. The function $f_{\text{air}}(x)$ is therefore a mathematical convolution of $f_s(x)$ and $\phi_{\text{opt}}(x)$, and $F_{\text{air}}(N)$ is a product of $F_s(N)$ and $F_{\text{opt}}(N)$. The function $\phi_{\text{opt}}(x)$ gives the image intensity of an infinitely long and infinitely narrow line intensity, when only the optical system is used.

If a photographic emulsion is placed in the plane of the aerial image, then the light distribution will be changed because of scattering and rescattering by the emulsion particles and the silver halide grains etc., and because of absorption in, reflection from and transmission through the emulsion layer. The geometrical diversions of the latent image are therefore larger than the optical aerial image. Upon developing, the photographic emulsion is affected chemically in such a way that a silver image appears more or less strongly depending on the effectiveness of the developer. This transparent image can be represented by means of the $I \cdot t$ values, calculated from the density by means of a density curve. It is this fictitious intensity image which is called $f_i(x)$ and defined as the effective image. The Fourier transform belonging to this image is $F_i(N)$. Thus, by light distribution function $\phi_{\text{em}}(x)$, we mean the fictitious image intensity which is obtained in an emulsion when an infinitely long and infinitely narrow line intensity is exposed. The corresponding CT function is $F_{\text{em}}(N)$. The convolution between $\phi_{\text{opt}}(x)$ and $\phi_{\text{em}}(x)$ is called $\phi_{\text{tot}}(x)$ and the corresponding transfer function

$$F_{\text{tot}}(N) = F_{\text{em}}(N) \cdot F_{\text{opt}}(N) \quad (10)$$

Among the test objects which can be used in an investigation of this type, the sinusoidal line pattern possesses the great advantage that its structure and shape are preserved by the photographic process. The sinusoidal pattern also possesses advantages from a purely mathematical point of view. Hence, for these reasons, a periodic, sinusoidal line pattern was used in this investigation. The lines were arranged in groups with an increasing frequency (Fig. 1). The period length within each group was constant. All of the periods in the line pattern had the same amplitude.

A small number of frequencies is sufficient to describe the behaviour of $F_{\text{em}}(N)$. The period length is preserved right through the combination of optical system and emulsion (except for the scale which depends of course on the optical system). The amplitude, however, decreases as the distance between the lines decreases. The

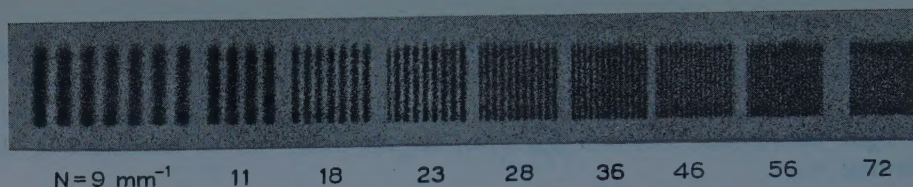


Fig. 1. Photographic reproduction of a periodic sinusoidal line pattern. The lines are arranged in groups with an increasing frequency. Within each group, the length of the period and the amplitude are constant.

lines are of constant intensity in the y direction and are sufficiently long. However, at right angles, i.e. in the x direction, the intensity varies sinusoidally. The pattern possesses therefore a one-dimensional character.

That the period length and sinusoidal shape are maintained during photographic reproduction can be expressed mathematically [13]. The condition is that the frequency functions included in the system are linear, i.e. the amplitude of the output wave is directly proportional to the input wave. Assuming that the line pattern of the object can be described by means of the function:

$$f_o(x) = 1 + C_o(N_k) \cdot \sin 2\pi N_k x, \quad (11)$$

the image of the photographic emulsion will be described by:

$$f_i(x) = 1 + |F_{\text{tot}}(N_k)| \cdot C_o(N_k) \sin [2\pi N_k x - \Theta(N_k)] \quad (12)$$

where $|F_{\text{tot}}(N_k)|$ is the CT of the optical system and photographic emulsion together. This CT function will describe how the amplitude diminishes. $\Theta(N_k)$ is the phase shift which, in this investigation, has been neglected because of the symmetry of the image.

Apparatus for the measurement of $|F_{\text{tot}}(N)|$

Points on the $|F_{\text{tot}}(N)|$ curve can be determined by exposing a sinusoidal intensity distribution on to a photographic emulsion. The apparatus constructed for this purpose has therefore two main parts, viz. a light modulator for regulating the amount of light and an optical arrangement for exposing an object.

The optical arrangement consisted of a collimator, mirror and a camera lens (Fig. 2 a). A slit S (width 10μ , length 18 mm) was placed in the focal plane of the collimator K (focal length 938.5 mm). The slit was sufficiently long that diffraction phenomena in the y direction could be neglected. The parallel beam of light proceeding from K was reflected by a plane, aluminized mirror M . It then passed through a camera objective C (focal length 25 mm) and formed a line element on the film emulsion. The line element was therefore an image of the slit, situated at an infinite distance relative to the objective. Upon slowly turning the mirror M (Fig. 2 a, Fig. 5), the line element moved over the emulsion surface in the x direction and thus produced a rectangular image. However, if the slit intensity simultaneously varies sinusoidally with respect to time (Fig. 3):

$$I(t) = I_0 + I_A \sin \omega t \quad (13)$$

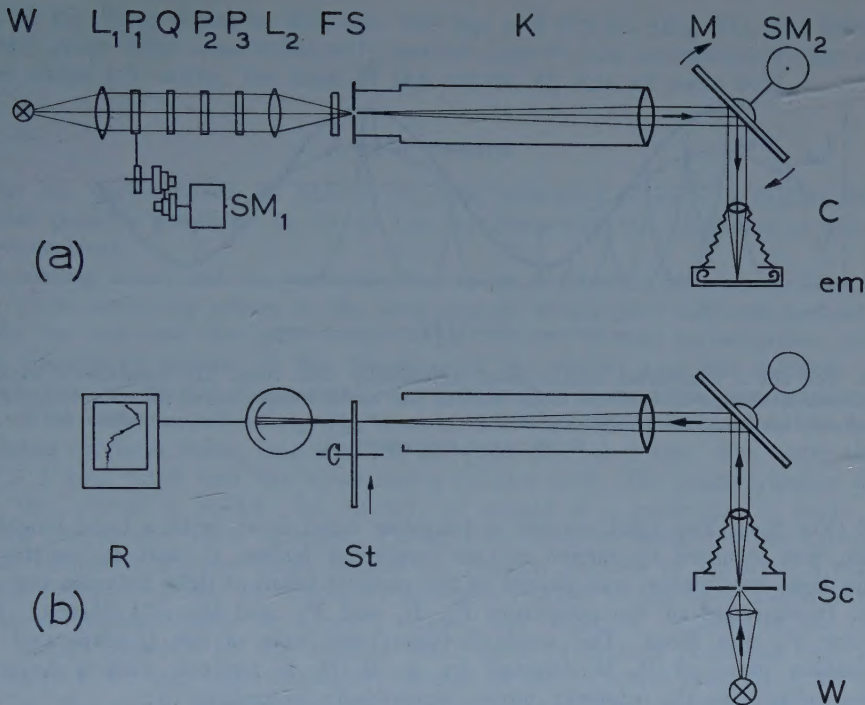


Fig. 2. Apparatus obtaining contrast transfer functions.

(a) Arrangement for combination of optical system and photographic emulsion; W , tungsten band lamp; L_1 and L_2 , lenses; P_1 , rotating polarizer driven by synchronous motor SM_1 and gearbox; Q , quarter-wave plate; P_2 and P_3 , polarizers with planes of polarization fixed at angles α and β to an axis of Q ; F , filter; S , entrance slit of the collimator K ; M , mirror rotated slowly by synchronous motor SM_2 ; C , camera lens; em , photographic emulsion.

(b) Arrangement for optical system alone: W , tungsten lamp; Sc , narrow entrance hole in the plate holder plane of the camera C which is to be tested together with the mirror and collimator; St , rotating Siemens-star; R , recorder connected to the photomultiplier cell at the end of the collimator.

then a line pattern which is a function of the coordinate x' will be exposed on the emulsion:

$$I_{\text{image}} = I'_0 + I'_A \sin 2\pi N x' \quad (14)$$

Hence, such a sinusoidal line pattern corresponds to an object of the same type. The whole of the exposure did not occur simultaneously. This type of reproduction permitted, namely, the moving light image to be summed over the whole of the exposure period [7].

The contrast transfer function is the ratio of the contrast of the image and that of the object. Fig. 3 shows, for example, that a decrease of the contrast 0.8 of the aerial image to the contrast 0.4 of the effective image gives the value $|F_{\text{em}}(N)| = 0.5$.

The sinusoidal variation (with respect to time) of the slit intensity was produced by means of a light modulator placed between the light source W and the

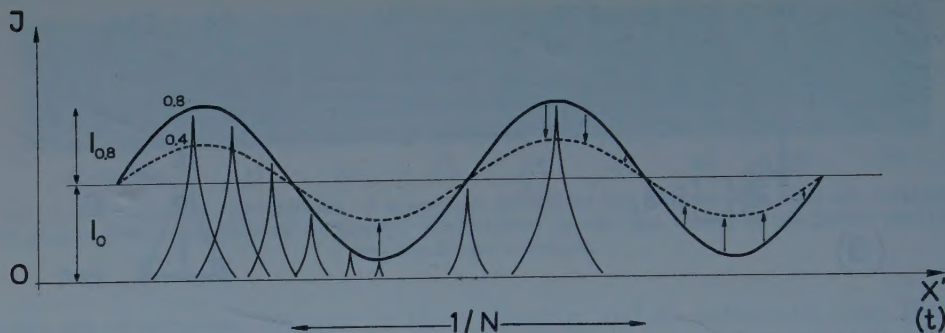


Fig. 3. The slit is modulated and exposed sinusoidally with time. The emulsion is exposed as abscissa. A change in contrast from that of the aerial image, i.e. of the object only (0.8), to that of the image on the emulsion (0.4) gives $|F_{em}(N_k)|$ for the emulsion = 0.5 for the fixed space frequency N_k .

slit S (Fig. 2 a). The light source, a tungsten band lamp with a band length of 20 mm, was focussed by means of two condensor lenses, L_1 and L_2 , on the slit S . The light modulator was placed in the parallel beam of light between the two lenses. It consisted of the polarizers P_1 , P_2 and P_3 , and the $\lambda/4$ plate Q . The polarizer P_2 was fixed. The angle between one axis of the Q plate and the polarization plane of P_2 is denoted by α . If P_1 is rotated with a constant angular velocity ω , the intensity varies sinusoidally according to:

$$I(t) = I_0 \cos^2 \beta (1 - \cos 2\alpha \cdot \cos 2\omega t) \quad (15)$$

Each rotation $\omega \cdot t = 2\pi$ gave rise to two density maxima on the emulsion em . Between 0.5 and 4 sinusoidal periods per sec were produced by means of a synchronous motor SM_1 and nine-speed reduction gear. The synchronous motor SM_2 was equipped with a gear box which produced three different speeds of rotation for the aluminium mirror M . Hence, 27 different space frequencies from $N = 9 \text{ mm}^{-1}$ could be obtained.

The contrast of the object is determined by the angle α . It is therefore possible to arrange the experiment so that the intensity variation only extends over a part of the density curve. For example, a suitable part of the density curve around the "working point" (i.e. the density D which corresponds to the mean intensity of the sine wave) can be investigated. The position of the "working point" can be set accurately by rotating P_3 through the angle β in relation to P_2 . However, with high speed films, a grey filter must be placed in front of the slit for the rough adjustment of the intensity in order to facilitate the accurate setting of the absolute intensity by means of the rotation of P_3 .

The spectral range was limited by means of a filter F (Fig. 2). The combination of tungsten band lamp and filter gave a sufficiently high intensity for emulsions of low sensitivity and also a sufficiently narrow spectral band for the investigation of the colour sensitivity of the emulsion. Since the $\lambda/4$ plate Q was designed for $\lambda = 546 \text{ m}\mu$, a wide filtering of the light would mean that full contrast $C_0 = 1$ would not be obtained. Three spectral regions have been investigated with the

$\lambda_{\max}$ of the filter equal to $456\text{ m}\mu$, $546\text{ m}\mu$ and $679\text{ m}\mu$ (Fig. 21). The filtration by the green filter was sufficiently narrow that C_o had approximately the maximum value 1.0, while, for both of the others, C_o was at most 0.94.

The camera

For an investigation of this type, it is necessary to have a camera of the highest possible precision as far as the focussing and the planeness of the film are concerned.

Focussing could not be performed by using a screwing motion for the objective since centering errors in the lens system would give different optical CT values for one and the same image angle. Hence, in this investigation, only a pure translatable motion in the direction of the optical axis was suitable.

The lens holder on the camera was constructed from three concentric cylinders. The outer was fixed to the camera housing. The middle one, which was threaded on both sides, was situated between the other two. The outer thread had a 1 mm pitch and the inner one a 0.5 mm pitch. The inner cylinder, which was the objective holder, was forced, by means of a guide-pin, to move in a groove cut in the outer cylinder. The groove therefore restricted the movement of the objective to one single degree of freedom parallel to the axis. A turning of the middle cylinder in a positive direction relative to the outer stationary one produced automatically an equally large, but negative, rotation of the objective holder relative to the middle one. Each revolution produced thus automatically a 0.5 mm pure translatable motion of the objective relative to the emulsion in the same direction as the original motion. The combination of a high precision in the making of the threads and a large number of threads, which grip on to each other, made possible a high reproducibility in the focussing (about $2\text{ }\mu$). This has been checked by the many adjustments in which check recordings of $|F_{\text{opt}}(N)|$ in the image space of the lens system have been made. As an interesting comparison, it should be mentioned that the thickness of the emulsion Agfa IFF is ca. $6\text{ }\mu$ while that of Agfa IU is approximately the double.

To prevent the bending of the emulsion layer, the film was pressed between two accurately worked plane surfaces which had been controlled interferometrically. In the front plate, a small opening ($3 \times 12\text{ mm}$) was left free for the passage of light.

Measurement of the optical CT function $|F_{\text{opt}}(N)|$

In order to determine the CT function $|F_{\text{em}}(N)|$ for the emulsion accurately, all of the elements between the object and the image which decrease the contrast must be known so that they can be eliminated. An auxiliary investigation has therefore been made in order to determine the contrast transfer function for the lens system, $K + M + C$, by means of a photoelectric method. The principle and some details concerning the apparatus have been described previously [9], [7].

The testing of the optical system of the apparatus ($K + M + C$) (Fig. 2 b) was performed in such a way that the arrangement of the optical system was identical with that used in the experiments on the emulsion. A light point ($1.5 \times 2.2\text{ }\mu^2$) was reproduced by the lens system in the image plane of the camera on a rotating star situated in the focal plane of the collimator. The light path was therefore the same as that in the emulsion experiment but the technical difficulties

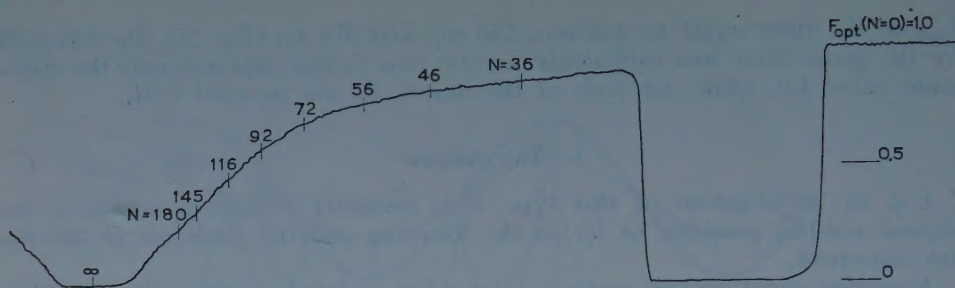


Fig. 4. A registration showing a *CT*-curve for the optical system. A calibration level for $F_{\text{opt}}(0) = 1$ and $N = 0$ is to the right. On the left hand side is shown an uncorrected $|F_{\text{opt}}(N)|$ curve for $N > 30 \text{ mm}^{-1}$. The space frequencies used for the investigation of the emulsion are shown as dots. Their distance from the ∞ position is obtained from $r = \frac{\text{constant}}{N}$.

necessitated the different light direction. The adjustment of the test star in the focal plane of the collimator is, amongst other things, less critical and a variation of a few μ during the rotation is without appreciable effect on the results. The ground frequency of the light stream which passed through the star was amplified selectively. The alternating current signal was rectified and the contrast of the image registered on the recorder *R*. The star was moved radially and a *CT* curve was recorded simultaneously on a relative scale. In order to calibrate this scale, the test star *St* with its finite radius was moved outside the light path. Instead, the light stream was modulated quadratically with the same frequency using an extra rotating sector so that the space frequency had the value $N < 0.03 \text{ mm}^{-1}$. This vertical calibration could be performed according to $F_{\text{opt}}(0) = 1$ [6]. The registration in Fig. 4 shows the calibration on the right and the $|F_{\text{opt}}(N)|$ curve on the left—the frequencies chosen for the experiment are marked on the experimental curve. A radius of the star corresponds to a distance r from the axis of symmetry of the curve (marked by ∞), since the middle point of the star passes the point image. The position of N is determined by

$$r = \frac{m \cdot G \cdot v_s}{2\pi \cdot v_t} \cdot \frac{1}{N} = \text{constant} \cdot \frac{1}{N}$$

m = number of spokes in the star ($m = 70$)

G = enlargement of the optical system

v_s = registration speed of the recorder

v_t = radial translatory velocity of the star.

However, a correction must be applied since the finite dimensions of the light source influence the measurements. The deflection of the recorder $|F_{\text{reg}}(N)|$ gives a lower absolute value than the correct $|F_{\text{opt}}(N)|$. If the slit function is denoted by $F_{\text{sc}}(N)$, then we obtain by division with the real transform

$$F_{\text{opt}}(N) = \frac{F_{\text{reg}}(N)}{F_{\text{sc}}(N)} \quad (16)$$

$F_{\text{sc}}(N)$ is determined after taking consideration of the fact that intensity distri-

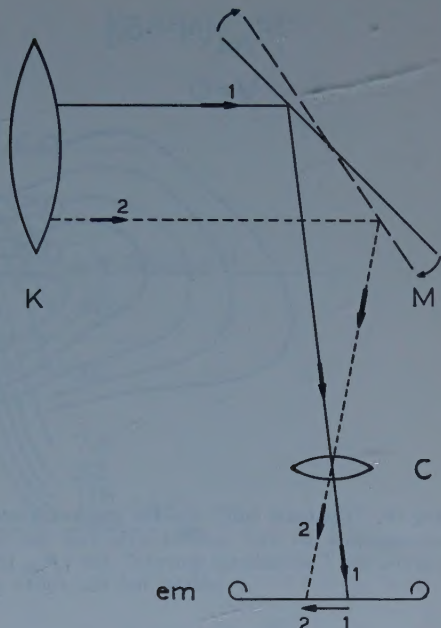


Fig. 5. A rotation of the mirror *M* produces a movement of the image on the photographic emulsion *em*. Rays of light, which pass through the camera lens *C*, come from different parts of the collimator lens *K* and the mirror *M*. Hence, the optical system must be tested in the experimental arrangement using exactly the same light path.

bution of the point is almost rectangular. Its influence is described by the Fourier transform:

$$F_{sc}(N) = \frac{\sin \pi N b}{\pi N b} \quad (17)$$

When recording the $|F_{opt}(N)|$ curves, the hole was placed so that the side $= 2.2 \mu$ was in the direction of translation of the test star. The value was very difficult to determine exactly, but the accuracy was sufficient for these investigations. Moreover, the frequencies were not so high that it was necessary to employ the steeper parts of the correction curve. $N < 180 \text{ mm}^{-1}$.

$|F_{opt}(N)|$ shall also contain a correction for the finite width of the slit *S* which was used for the emulsion tests. This correction is, however, ≈ 1 with a difference less than 0.5 % for $N < 180 \text{ mm}^{-1}$. In relation to the other sources of error, this one was completely negligible.

$|F_{opt}(N)|$ changes slowly when a movement in the emulsion plane *xy* is made in the image space of the camera. Fig. 5 shows how a turning of the mirror *M* produces a movement of the image on the emulsion from position 1 to 2. Each *N* region, viz. $N_1, N_2, \dots$, necessitates its own image angle. Hence $|F_{opt}(N)|$ must be determined for the different image angles. The method for accurately determining the position of the emulsion, which is described below, requires a knowledge of $|F_{opt}(N)|$ for different planes parallel to the emulsion.

The recording of the sinusoidal pattern in the emulsion tests was performed for the extension of the line in the *y* direction. The line length ($\sim 0.4 \text{ mm}$) corresponded to an angle $< 0.9^\circ$. It was therefore sufficient with a mapping in the *xz* plane with the lens system in the same position as when exposing the emulsion. When measuring the optical *CT* function, the depth adjustment along

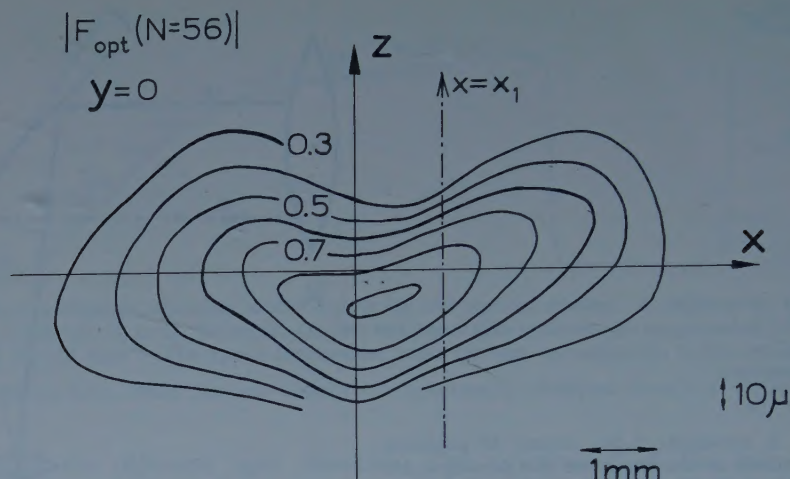


Fig. 6. "Contrast hill" for the optical system in the xz plane of the image space. The z axis corresponds to the optical axis and the x axis to the position of the emulsion plane. The curves are "isocontrast curves" for $|F_{\text{opt}}(N=56)| = 0.3, 0.4, \dots$. The contrast curves are corrected for the finite extension of the test point.

the z axis was varied in 10μ steps for each image angle adjustment. The image angle was set along the x axis in 1 mm steps. The measurements were performed for space frequencies from 9 to 180 lines/mm. They are given in the CT registration in Fig. 4. Each frequency is topographically prepared as a function of the depth adjustment z and the image coordinate x (Fig. 6). Lines with the same contrast, so called isocontrast lines, give a "contrast hill" of great breadth but small depth. The figure shows the contrast hill for $N = 56 \text{ mm}^{-1}$. The best contrast reproduction was obtained in the neighbourhood of the optical axis. The contrasts are corrected numerically for the finite width of the point. The z axis corresponds to the optical axis of the lens system in the image field. However, the figure shows that the notion of "optical axis" cannot strictly be defined for optical systems with centering errors. This appears even more clearly for "contrast hills" corresponding to higher N values. When the number of lines is increased, the "contrast hill" tends to approach the objective somewhat. The tendency becomes greater the further one moves from the central region. The "contrast hill" becomes at the same time lower and less spread out. This appears clearly from a section taken through all of the "contrast hills" along $x = x_1, y = 0$ (Fig. 7). By means of a suitable placing of the emulsion, it is possible to measure the CT value for relatively high frequencies. The dimensions of the test star and the CT properties of the emulsion limit the usable frequency range to 36–180 lines/mm.

An interpolation procedure was used for the space frequency range $N < 36 \text{ mm}^{-1}$. This is permissible since an interpolation is both obvious from a theoretical point of view and also practical because the points (x_k, z_k) of good contrast in the region, $36\text{--}100 \text{ mm}^{-1}$, fit on a straight line passing through $F_{\text{opt}}(0) = 1$. In Fig. 8, the CT curves are given for the points (x_k, z_k) . The interpolated values for

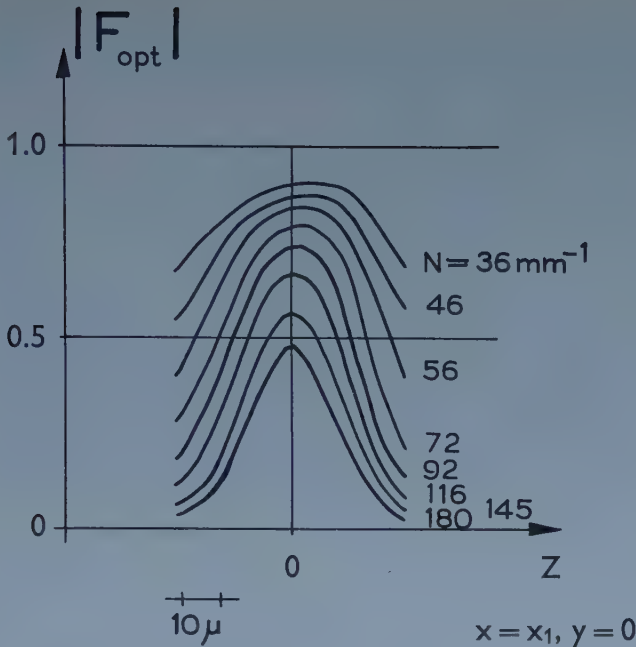


Fig. 7. Section through all the contrast hills at $x = x_1, y = 0$. The function $|F_{\text{opt}}(N)|$ has been corrected.

the regions exposed with the space frequencies $N_k = 9, 11, 18, 23$ and 28 mm^{-1} are marked.

The ideal experimental arrangement would be to replace the rotation of the mirror by a very accurate translation of the emulsion. When the mirror is removed, the light path can then be straightened out. The CT testing of the lens system would only need to be performed axially and could therefore be considerably simplified.

Demands on the optical system

Considerable demands must be placed on the optical system when measuring photographic CT values for space frequencies N up to $150\text{--}200$ lines per mm. Amongst other things, the system must be able to produce sufficiently high and known contrasts in the emulsion plane (cf. in this connection Ronchi [11]).

An objective with a short focal length has in general higher CT functions than those given by objectives with long focal lengths. A microscope objective certainly gives high values but possesses instead the disadvantage that it produces good contrasts only within a small part of the image field. Difficulties occur when trying to place the emulsion in a reproducible position. The thickness of the emulsion therefore has a considerable effect on the results. Also, the width of the focal plane is not sufficient for the type of experimental arrangement used here. It is possible to obtain a few individual measurements but not the long series of values on which accuracy in this work is necessarily based.

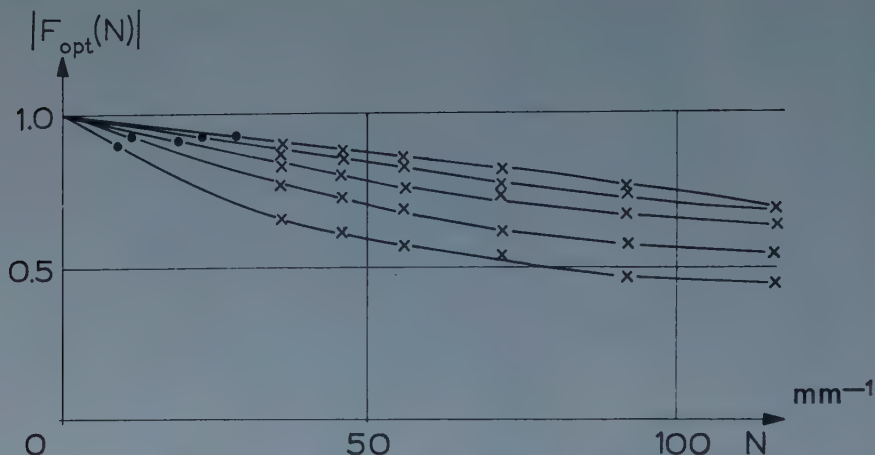


Fig. 8. *CT* curves determined at various image angles in one of the emulsion investigations. $\times$ are the experimentally determined points obtained from the contrast curves, $\bullet$ gives $|F_{\text{opt}}(N)|$ for $N < 36 \text{ mm}^{-1}$ and are interpolated values.

A wide aperture lens is necessary if less sensitive emulsions are to be investigated. Smaller apertures are not so satisfactory from the point of view of contrast as $F_{\text{opt}}(N)$ has a lower absolute value. For the maximum aperture, the deterioration in the *CT* value puts a limit to the usefulness of such an objective. This has been shown by investigations at, amongst other places, this Institute. As a compromise, the $f/2$ aperture of a cine objective, viz. $f/1.5$ Schneider Xenon, $f = 25 \text{ mm}$, has been chosen for all the experiments. Such an aperture is relatively large and gives, in addition, good *CT* functions. The other components in the lens system, viz. the collimator K and the mirror M , must of course also be of good quality. They are however of less importance in this connection:

Demands on the developing procedure

In these investigations, it was important to prove that the measurements of the light distribution in the sensitive emulsion layer were not adversely affected by an ineffective stirring of the developer at the emulsion surface. In stationary bath development, the chemical processes cannot alone produce sufficient stirring to prevent the occurrence of concentration gradients of developer components and chemical products. Such gradients produce uneven development (adjacency effects), which is of course undesirable. An irregular stirring of the developer is therefore necessary. However, a standard spiral developing tank, in which an irregular manual stirring is employed, is also unsuitable since the thin capillary liquid layer close to the emulsion surface is not stirred sufficiently effectively by the movement of the film in the developer. Thus, for certain combinations of emulsion and developer, a considerable adjacency effect is obtained. It is however not correct to state that this effect is, in general, quite undesirable under normal circumstances. However, it is obvious that the present investigation should be free from all such side effects. A third type of development, viz.

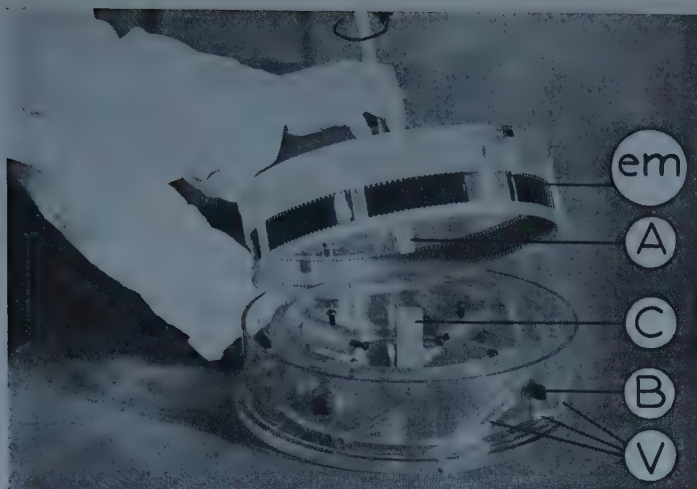


Fig. 9. Tank for brush development. The film holder *A* is turned by means of a synchronous motor causing the four weasel hair brushes *B* to pass lightly over the emulsion *em*. *C* is a bearing for the film holder axle. The developer, which tends to rotate together with the film, is made to flow irregularly by means of the partitions *V*. The tank is surrounded by thermostated water.

brush development, has however given acceptable results. The brushes, which only touched the emulsion very lightly, produced an effective stirring of the capillary layer without damaging the emulsion. Undesirable concentration gradients were reduced to a minimum as far as could be judged from the results. A tank for brush development has been constructed. As also shown by, amongst others, Clark [1], brush development possesses several advantages, viz. the development is very even and, moreover, reproducible. It should also be pointed out that brush development has been advised in Germany for photographic sensitometry (according to DIN 4512).

The tank (Fig. 9) was made of perspex. It was not made light-tight because the films were developed in a darkroom. The film *em* lies in a circle with the emulsion side inwards. The maximum length of 35 mm film which can be developed is 80 cm. The tank is built in two parts. The fixed lower part, which is placed in a thermostat containing vigorously-stirred water, consists of a cylindrical, ring-shaped vessel containing 1 litre of developer. Four soft, long and flat weasel-hair brushes were fixed to the inside of the vessel so that they touched the emulsion surface very lightly. The movable part, i. e. the film holder, was also cylindrical. It was constructed so that the film could be pushed into two grooves and then securely fastened. The axle *A* of the holder fitted into a perspex bearing. A synchronous motor fixed to the axle rotated the film holder with a radial velocity of 12 r.p.m. A brush therefore sweeps over each part of the emulsion every 1.25 sec. When the holder rotates, the developer also tends to rotate. However, the brushes prevent this and thus produce an irregular motion of the liquid. The motion of the developer is made even more irregular by means of obliquely-placed plates *V* fixed to the bottom and walls of the outer vessel. The stirring thus produced is very effective.

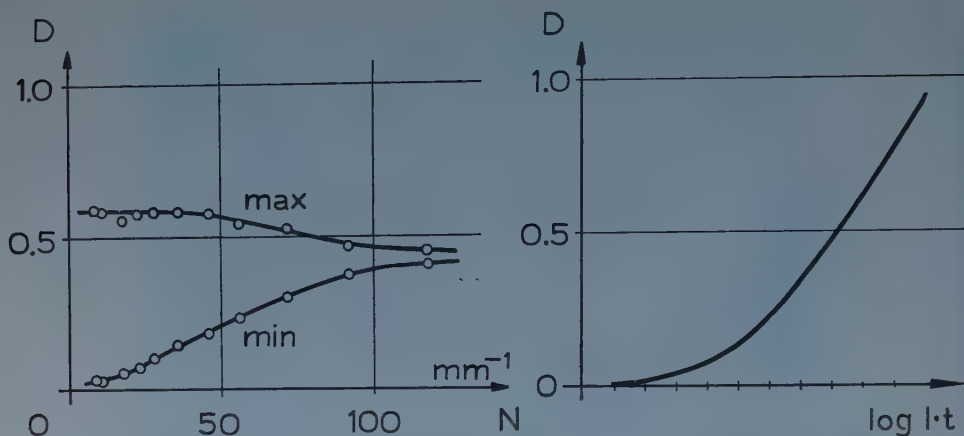


Fig. 10. Density maxima and minima of the sinusoidal line pattern in the negative as a function of the space frequency N . The "working point", i.e. density for the mean intensity, is $D = 0.45$.

Measurement of $|F_{\text{tot}}(N)|$ and calculation of $|F_{\text{em}}(N)|$

The image angle which must be used when exposing more than about ten of the space frequencies is determined by their "contrast hill". The exposure range is chosen after taking consideration of several factors. It is not possible to use always the region with the highest $|F_{\text{opt}}(N)|$ levels. The region which is suitable for the exposure of low space frequencies is much wider than that for high frequencies since the latter consists of narrower and shorter "contrast hills". The "contrast hill" becomes more bent and, at the same time, more displaced towards the objective as the frequency increases. The displacement varies moreover with the image angle. As a compromise, the plane of the emulsion was chosen so that it passes through the maximum of the contrast hill at $N = 56 \text{ mm}^{-1}$.

The method used in this investigation makes it possible to perform diffusion investigations on different parts of the density curve. Both weak and high densities give rise to disadvantages from a practical and experimental point of view, e.g. disturbances due to noise caused by non-statistical grain distribution, difficulties with the transmission measurements etc. The exposure was chosen so that, in all experiments, a density $D = 0.45$ was obtained. This density corresponded to a mean value of the object intensity of mainly the higher space frequencies (Fig. 10). A desire to place the working point immediately above the toe of the density curve, i.e. on the lower part of the linear section of the curve, where the best image reproduction is obtained has been fulfilled [8]. The figure shows the density of the image of the sinusoidal pattern obtained with an Agfa IFF film as a function of the frequency N . The modulation degree C_0 of the object was 1.0. The upper curve shows the maximum density and the lower the minimum density of the sine curves of the image. The corresponding density curve is given on the right.

As well as the exposing and developing of the emulsion, the experimental work also involved the microphotometrical investigation of the emulsion and the evaluation of the photometer curves.

The microphotometry of the emulsion was performed using parallel light. The microscope objective, $N.A. = 0.25$ and $f = 25$ mm, enlarged the emulsion 12.8 times on to a slit. The linear velocity of the emulsion was 150μ per minute. The effective photometer slit was 0.3 mm long and 0.7 – 1.5μ wide. An increase in the noise in the emulsion necessitated an increase in the width of the slit. The light stream, which was obtained from a photomultiplier tube placed behind the slit, was registered on a recorder in the form of a transmission for the negative.

The transmission measurements were made relative to the background fog of the emulsion, i.e. the transmission of the background fog was put equal to 1.0. The CT values thus calculated were therefore somewhat higher than those measured relative to the film base or "air". But since the contrasts should be expressed using quantities which have been measured using the effective illumination $I \cdot t$, it seemed most natural to use the CT value relative to the background fog. In order to, if possible, describe more completely the emulsion experiments, it would be best to state the magnitude of the background fog as well as the CT values.

In order to transform the photometrically measured transmission of the negative to the actual intensity $I \cdot t$, it was necessary to expose also a density strip. In this way, the influence of a non-linear density curve could be eliminated. The procedure is as follows. The transmissions for the maxima and minima of the photometric curve were transformed to $I \cdot t$ values by means of the density curve (Fig. 11). The contrast $|F_{\text{tot}}(N_k)| \cdot C_o(N_k)$ in equation (12) for the effective image was calculated using equation (4). Division of the contrast of the image by the $C_o(N_k)$ of the object (eq. 11) gave a point $|F_{\text{tot}}(N_k)|$ on the combined transform for the optical system and emulsion $|F_{\text{tot}}(N)|$. By means of a point by point division, $|F_{\text{em}}(N)|$ was calculated for the light distribution of the emulsion (eq. 8). Between 10 and 14 points were investigated for the $|F_{\text{em}}(N)|$ curve.

But now the question arises: "How is the sinusoidal shape changed by the emulsion?" The preservation of the sine shape is one condition for making calculations using contrast transfer functions. A density curve is shown in Fig. 11. One period of the registered photometric curve, with the transmission measured relative to the background fog, is plotted at the T axis. Below, one can find, at the $I \cdot t$ axis, the corresponding sine curve in $I \cdot t$ values. If a number of points from a sine curve in $I \cdot t$ values are transformed to the corresponding transmission values and are thereafter fitted into an original registration, then it is found that the points agree well with the registration (Fig. 12). Of course, local deviations occur depending on the noise. But the agreement is unmistakable. The figure shows the most unfavourable case, viz. the contrast of the image is so high that a large part of the density curve must be used for the evaluation. But no appreciably higher frequencies, apart from those due to the noise, were found to be imposed upon the ground frequency of the sinusoidal pattern of the effective image. For the high frequencies of the effective image the contrast is rather low so an almost straight part of the density curve is used. The density curve of the negative has for these frequencies nearly a sine form. The correctness of the structure of the calculated image in equation (12) was thus experimentally verified. The linearity was also proved later by the agreement of the results.

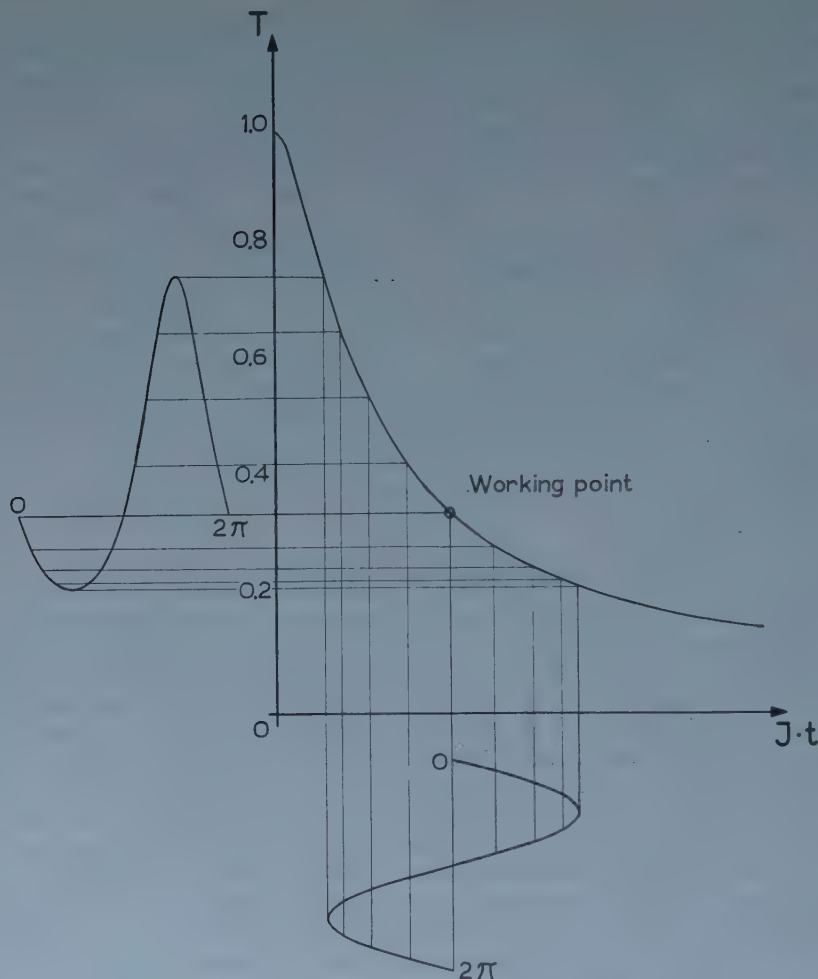


Fig. 11. The density curve for the IFF-emulsion shows transmission T as a function of $I \cdot t$. A period on the negative, which has been altered due to the density, is plotted at the T axis. The corresponding $I \cdot t$ curve has a sinusoidal form and is shown at $I \cdot t$ axis.

Influence of the positioning of the film

Series of exposures were made in order to show that the determination of $|F_{\text{em}}(N)|$ is independent of the position of the emulsion plane. The accuracy of measurement was obtained from a series obtained using the Agfa IFF emulsion B. The emulsion in the xy plane was exposed with $N = 56 \text{ mm}^{-1}$ within the whole of the image field corresponding approximately to the extension of the "contrast hill". The exposure was repeated a number of times on one strip of film using a distance of 10μ between the xy planes. Twelve sections with $x = \text{constant}$ were chosen. $|F_{\text{tot}}(N = 56)|$ and $|F_{\text{opt}}(N = 56)|$ were determined for each section and

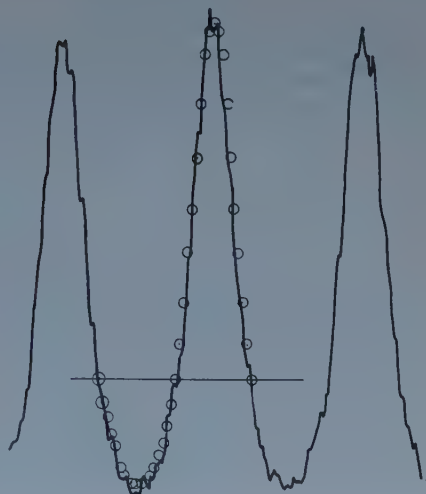


Fig. 12. A transmission curve constructed from a sinusoidal pattern is fitted in to an original registration. $N=9 \text{ mm}^{-1}$. The construction is performed with the aid of a density curve. The points indicate the constructed values for every tenth degree of a period.

exposure. The maxima of corresponding section curves were moved so that their z coordinates coincided (Fig. 13). $|F_{\text{em}}(N)|$ was calculated for each exposure point. The mean value $|F_{\text{em}}(N=56)|$ of 60 measurements was 0.607 and the Gaussian standard deviation 0.037.

The relative horizontal level of the $|F_{\text{em}}(N)|$ curves, together with the small standard deviation, confirmed that the transform of an emulsion, like the one obtained here, is real and that the influence of a change in the focussing has been eliminated.

The film investigation

The camera was constructed primarily for 35 mm film. When choosing film emulsions for the investigation, it was considered that a close similarity of the chemical compositions of the fine grain and coarse grain emulsions was desirable. However, because of insufficient information concerning the various emulsions, this has unfortunately not been possible—it was instead necessary to use photographic emulsions from one and the same manufacturer. It was however possible to satisfy the condition that all films should be developed using the same developer. In this way, the number of factors exerting an influence on the results has been kept to a minimum. The tests were made using the Agfa emulsions IFF, IF, ISS and IU. Brush development using the trade developer "Final" was employed. Frieser has also investigated the same emulsions in his diffusion investigations [4].

In all experiments, the mean density was 0.45 (Fig. 10). Each experimental curve value was an arithmetical mean of at least three measurements, each comprising between 4 and 30 periods. In addition, three exposures of a density strip were always made on the same piece of film.

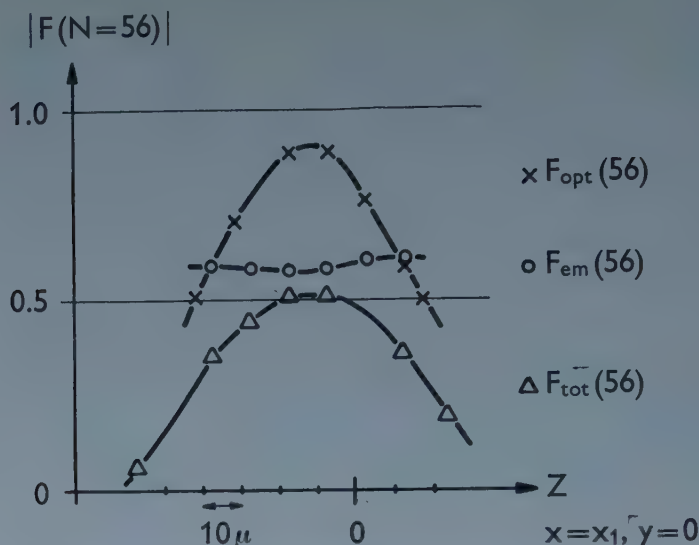


Fig. 13. Section through the contrast hills along the z axis for the frequency $N = 56 \text{ mm}^{-1}$. The function $|F_{\text{em}}(N = 56; z)|$ is calculated from the measured curves $|F_{\text{tot}}(N = 56; z)|$ and $|F_{\text{opt}}(N = 56; z)|$ according to

$$|F_{\text{em}}(N = 56; z)| = \frac{|F_{\text{tot}}(N = 56; z)|}{|F_{\text{opt}}(N = 56; z)|}$$

It was thereby shown that CT for the emulsion does not depend on the emulsion plane z or on the optical system. This demonstrates the homogeneity of the measurements.

Discussion of the microphotometric errors

The adjustment of the lines on the negative so that they are parallel to the slit is a source of error in the registration of the emulsion images. In order to investigate the accuracy of this adjustment, a single sine pattern has been registered 11 times with IFF emulsion B using a completely different adjustment each time. $|F_{\text{tot}}(N)|$ was calculated for five different frequencies (Table 1)—it represents five different contrasts. The standard deviation σ was calculated for each frequency. The amount of data is small. However, it is seen that σ tends to increase somewhat as N increases. The explanation is that the angular adjustment of the slit produces, in relation to the lines on the negative, increasingly greater error on

Table 1. Influence of the microphotometer slit.

N — space frequency;

M — mean value of $|F_{\text{tot}}(N)|$ for IFF emulsion B ;

n — number of registrations;

σ — standard deviation.

$N \text{ mm}^{-1}$	9	28	46	72	116
$M \%$	71	62	50	33	14
n	11	11	11	11	11
$\sigma \%$	0.8	1.2	1.5	1.2	1.7

the registered amplitude with increasing line frequency. On the whole, the standard deviations are very small and demonstrate the accuracy of the work. The correct adjustment should give a maximum amplitude for the sinusoidal scanning.

The finite width of the microphotometer slit causes the registration to be somewhat too low. A slit has namely the Fourier transform

$$F(N) = \frac{\sin \pi N b}{\pi N b}$$

Taking consideration to the background noise and the resolution at high contrasts, b can be put equal to one tenth of the period length for the highest frequency which is registered. The slit, which always has the same width, then registers all of the frequencies in the sine pattern. The transform indicates, at most, a 1.7 % lower value than that obtained with ideal scanning. The contrasts were often so small at higher frequencies that the error decreased to at most 0.003. With lower frequencies, the deviations from the correct values were smaller. Hence, errors resulting from the finite width of the slit could be completely neglected.

Since the emulsion was illuminated according to the Köhler arrangement with parallel light from an incandescent filament parallel to the lines in the sinusoidal pattern, it is possible to consider the illumination as being relatively coherent. The degree of coherence has not been determined. It was also not possible so far to determine a CT function for the microphotometer objective. There is however reason to believe that CT is rather constant for the frequencies between 0 and 150 mm^{-1} which were employed in this investigation. The theoretical boundary for incoherent reproduction with an objective for which $N.A. = 0.25$ lies at about 1000 mm^{-1} . A correction for the influence of the microscope objective has therefore, under such conditions, not been made.

An experimental analysis of the effectiveness of the electric recorder, including the inertia of the instrument, yielded a CT function which, at $N = 200 \text{ mm}^{-1}$, gave a CT value of 0.99. This influence could therefore be neglected.

Influence of the structure of the line pattern

In all emulsions, the emulsion grains form a mosaic pattern, the structure of which is more or less difficult to define. The structure presents a different appearance for different emulsions, depending on the differences in the grain size. A more sensitive emulsion has in general a larger mean grain size. The grains can exhibit a better statistical distribution the greater the area of the emulsion or the greater the number of grains. The distribution has a certain importance for the scanning of the test pattern. In order to get a more accurate and better result, the surface of the scanning slit should be increased as much as possible and should, at the same time, sweep over as large a region as possible. However, the size of the slit is limited by the highest usable line frequencies. A more accurate determination of the influence of the grains on the individual sine waves which make up the image is of a certain amount of interest for a knowledge of the degree of statistical adaptability of the grain. During the scanning of the pattern, the dimensions of the slit were kept constant. The contrast $|F_{\text{tot}}(N)|$

was measured and evaluated, period by period, within a number of N -regions. The arithmetical mean value M and the standard deviation σ was determined (Table 2). The number of periods depended on the line frequency. The table for

Table 2. Influence of the line structure.

M —mean value of $|F_{\text{tot}}(N)|$ for IFF emulsion B ;

n —number of periods;

σ —standard deviation;

$\frac{\sigma}{M}$ —standard deviation relative to the corresponding M value (%).

$N \text{ mm}^{-1}$	9	28	46	72	116
$M \%$	73	61	50	30	14
n	4	7	11	15	27
$\sigma \%$	1.1	1.6	2.2	2.4	2.2
$\frac{\sigma}{M} \%$	1.5	2.6	4.4	7.9	16

the IFF emulsion B shows how σ reaches relatively early its highest value. The explanation of this behaviour is as follows: If a low frequency, e.g. $N = 9 \text{ mm}^{-1}$, is considered, then the "sine curve" can be followed. The extreme values of the curve can therefore be determined relatively accurately (Fig. 12). Here, the grains are without doubt statistically distributed. But, for a fine structure, e.g. $N = 72$ lines/mm, it is not so likely that a normal distribution exists inside each extreme value. It is instead more probable that the mean grain size varies somewhat period by period. The deviations from the normal distribution within each half period will therefore be greater the finer the line structure. But, if the number of periods increases sufficiently rapidly, the number of periods will, together with a low M value, indirectly oppose the influence of the grain so that σ will no longer continue to increase. The ratio σ/M will however successively increase. It is not possible to perform any evaluation of the frequencies which are drowned by the noise obtained using the method employed in these experiments. For this, other methods than the one employed here must be used.

Variation of the contrast. Linearity

An exposure of the object according to equation (11) gives an effective image in which the form of the sinusoidal pattern is preserved. The contrast is, however, lower. Equation (12) shows the dependence of, in addition, $|F_{\text{tot}}(N)|$. However, linearity is said to exist only when the contrast of the image $|F_{\text{tot}}(N_k)| \cdot C_o(N_k)$ is directly proportional to the contrast of the object. This means that $|F_{\text{tot}}(N_k)|$ is theoretically independent of the magnitude of $C_o(N_k)$.

It is known, that linearity exists for the optical system. But this has not previously been verified for the case when a non linear part of the density curve is used. The investigation of this linearity is performed in such a way that the contrast $C_o(N_k)$ is varied. If, in the expression for the image contrast $|F_{\text{tot}}(N_k)| \cdot C_o(N_k)$, the function $|F_{\text{opt}}(N_k)| \cdot C_o(N_k)$ is to be proportional to the contrast

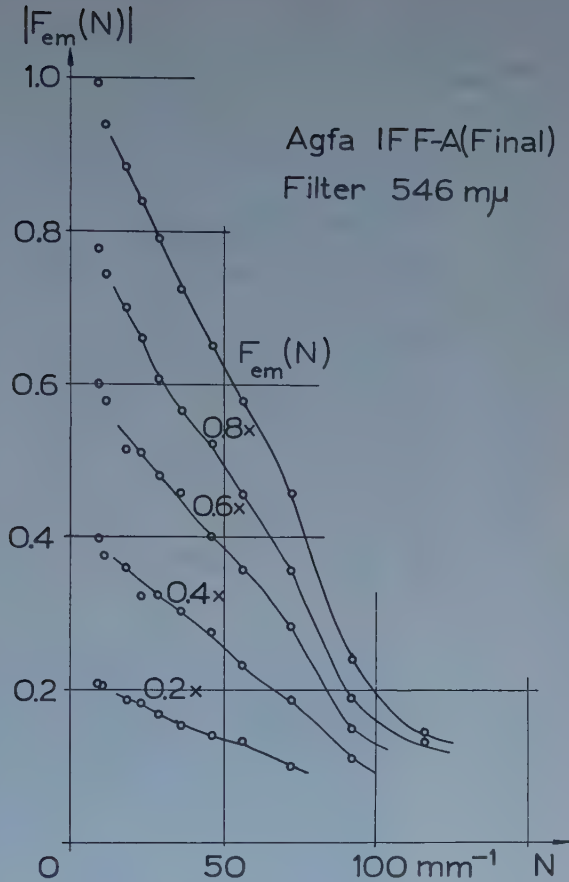


Fig. 14. The five curves represent $C_o(N) \cdot |F_{em}(N)|$. For the upper curve, $C_o(N) = 1.0$.

of the object, then $|F_{em}(N_k)|$ must be constant provided that linearity exists for the reproduction. It was, moreover, confirmed that $C_o(N_k)$ is preserved, just like $|F_{opt}(N_k)|$.

The results obtained, when $0.2 \leq C_o \leq 1.0$, are presented in curves and in a table. Fig. 14 gives the contrast transfer function $|F_{em}(N)|$ for the IFF emulsion A. The five curves show the behaviour of $C_o(N) \cdot |F_{em}(N)|$. For the uppermost curve, $C_o(N) = 1$. An evaluation of $|F_{em}(N)|$ alone, which is based on different C_o values, is presented in Fig. 15.

C_o is given here as the abscissa. Each curve gives $|F_{em}(N)|$ with N as a parameter. The slight sloping of the curves does not contradict the linearity of the photographic reproduction. There can be several reasons why $\frac{\Delta |F_{em}(N_k)|}{\Delta C_o} \neq 0$.

If there occurs, for example, a systematic error in the reading of the photometer curves for $C_o = 0.2$, then the error in the $|F_{em}(N)|$ curves will be enlarged by a factor of 5. Good results were also obtained for other commercial films (Table 3). The table only contains some of the frequencies comprising the contrast transfer functions. It gives a good idea of how the linearity is preserved. For $0.4 \leq C_o \leq 1.0$,

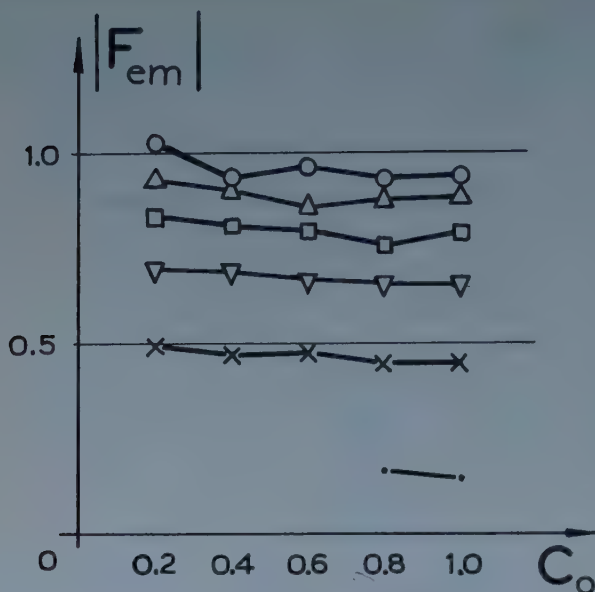


Fig. 15. The contrast of the object is the abscissa. $|F_{em}(N)|$ is the ordinate. ○, curve with the parameter $N=11 \text{ mm}^{-1}$; △, 18 mm^{-1} ; □, 28 mm^{-1} ; ▽, 46 mm^{-1} ; ×, 72 mm^{-1} ; ●, 116 mm^{-1} . The horizontality of the curves confirms that one can describe the change from applied intensity to effective intensity as a linear filter, i.e. the CT theory can be used in photography.

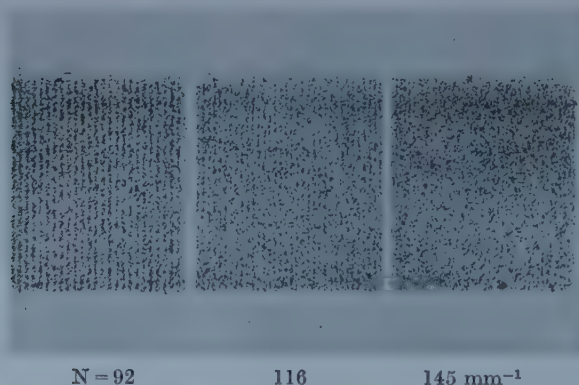


Fig. 16. At the "limit of resolution". The line pattern is obvious at $N=116 \text{ mm}^{-1}$. Higher space frequencies are resolved partially or not at all for $N > 116 \text{ mm}^{-1}$. The influence of the emulsion grain appears. IFF emulsion A. The contrast $C_0(N)=1.0$.

the variation for the various space frequencies are with a few exceptions quite small. A good linearity appears to exist. A Fourier analysis is permitted in experiments with different photographic emulsions in spite of the fact that relatively large inhomogeneities exist in the emulsion. If, however, the column $C_0=0.2$ is considered, one finds that the values sometimes exhibit deviations. Such deviations can be easily explained since, at these low contrasts, the noise, when the period length is small, causes a considerable disturbance of the sine waves. It is difficult to obtain accurate readings. At the same time, a systematic reading error, which affects the results, is easily introduced. According to the table, a coarse grained emulsion with a large noise has a larger effect on the sine wave. This

Table 3. $|F_{\text{em}}(N)|$ for various intensity contrasts C_0 applied to the slit S (in %).

Emulsion	N mm^{-1}	Contrast C_0 applied				
		1.0	0.8	0.6	0.4	0.2
IFF-A (Final)	11	94	93	96	93	103
	28	79	76	80	81	84
	46	65	65	67	69	70
	72	46	45	47	47	49
IF (Final)	11	94	96	99	90	92
	28	76	76	76	76	84
	46	58	58	57	54	69
	72	39	36	40	41	—
ISS (Final)	11	97	100	109	103	111
	28	65	69	73	77	88
	46	49	51	55	58	66
	72	37	40	44	49	—
IU-A (Final)	11	95	99	100	101	116
	28	56	58	67	61	58
	46	39	40	47	41	—
	72	26	30	35	—	—

result is also of importance as far as the method for performing the measurements is concerned. It is thus permitted to adjust the object contrast, when measuring, for example, the light distribution, so that C_0 becomes of a suitable value. Large image contrasts are namely disturbed by background fog and low ones by the noise. These investigations provide a new illustration of how problematical the notion of limit of resolution is for a photographic emulsion. The curves in Fig. 14 tend to pass slowly over the limit of resolution. The sine curves gradually die away and appear finally only within certain limited regions. Low image contrasts can be better by increasing $C_0(N)$. Moreover, the "limit of resolution" moves towards higher N values when a good objective is used. Fig. 14 and 16 show how, for the IFF emulsion, the pattern is obvious for $N=116 \text{ mm}^{-1}$ while the higher frequencies, e.g. $N=145 \text{ mm}^{-1}$, are not worth registering. If a curve $|F_{\text{em}}(N)|$, which has been obtained using a lower image contrast $C_0(N)$ e.g. 0.4, is employed, then the largest resolvable frequency will be smaller. When $C_0(N)=0.2$, the curve continues only to $N \sim 75 \text{ mm}^{-1}$.

Investigation of the different emulsion and emulsion batches

The investigation has been performed using four emulsions, viz. Agfa IFF, IF, ISS and IU. They represent different film speeds and different degrees of graininess. The CT functions for IFF, and IU have moreover been investigated for two different emulsion batches. The results are given in Fig. 17 and Table 4. Each value is the mean of 3–5 measurements. The maximum deviation from the mean value is also included in the table. The experimental values obtained confirmed that a fine grain emulsion gives in general a higher $|F_{\text{em}}(N)|$. For the frequencies $0\text{--}80 \text{ mm}^{-1}$, there occur namely large differences between the different curves, the fine grained IFF emulsion having the highest level. For

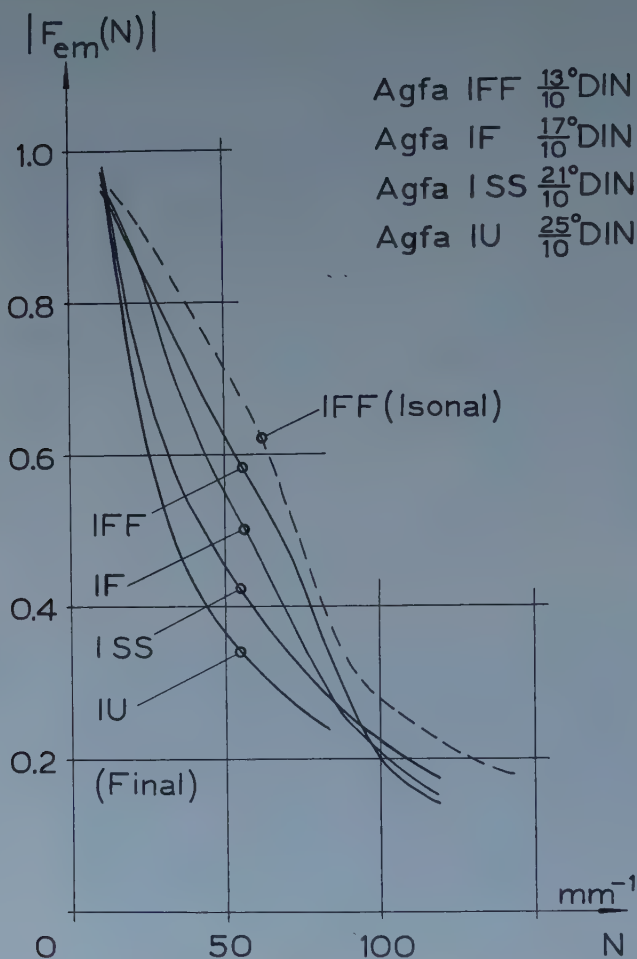


Fig. 17. $|F_{em}(N)|$ curves for four emulsions developed using Final. IFF in addition with Isonal. All emulsions, type A.

$N > 80 \text{ mm}^{-1}$, there often exists a considerable uncertainty because the influence of the grains becomes more dominant. The sine pattern appears more clearly with the finer grained emulsions in the high frequency region. With the IFF emulsion, the line pattern appears patchwise even at $N = 180 \text{ mm}^{-1}$. With the IU emulsion A, these patches have sometimes been observed at about $N = 100 \text{ mm}^{-1}$.

Questions which may be asked in this connection are: How large are the differences between two different emulsion batches of, for example, the IFF type? Does the CT function change when an emulsion is stored and thus aged? An investigation to provide answers to these questions would have to be performed over a relatively long period of time, preferably several years. No direct aging tests were performed. On the other hand, the CT functions for two emulsion batches, A and B, of both the IFF and IU types have been determined. The

Table 4.

$N \text{ mm}^{-1}$	$ F_m(N) \times 10^2$					
	IFF - A	IFF - B	IF	ISS	IU - A	IU - B
9	98 ± 0.7	103 ± 2.0	96 ± 0.7	109 ± 2.7	108 ± 1.3	117 ± 1.1
11	94 ± 0.3	99 ± 2.9	94 ± 5.9	97 ± 2.4	95 ± 3.7	109 ± 4.0
18	89 ± 0.7	98 ± 2.2	89 ± 3.1	81 ± 2.6	77 ± 1.9	89 ± 3.1
23	84 ± 0.6	94 ± 2.0	84 ± 2.6	71 ± 4.5	66 ± 0.1	82 ± 4.6
28	79 ± 1.6	87 ± 1.4	76 ± 5.5	65 ± 4.1	56 ± 2.5	68 ± 2.5
36	72 ± 1.1	80 ± 2.4	65 ± 4.7	54 ± 7.7	45 ± 1.8	59 ± 4.5
46	65 ± 1.3	70 ± 1.4	58 ± 4.8	49 ± 4.0	39 ± 1.2	49 ± 2.7
56	58 ± 2.3	60 ± 3.7	50 ± 3.7	41 ± 4.3	34 ± 3.1	38 ± 2.1
72	46 ± 0.6	51 ± 2.2	39 ± 1.7	37 ± 1.1	26 ± 1.2	33 ± 1.6
92	24 ± 0.7	40 ± 1.7	24 ± 3.3	25 ± 3.5		25 ± 3.0
116	14 ± 2.8	31 ± 1.7	16 ± 3.1	18 ± 2.6		18 ± 1.5
145		22 ± 1.4				

(All emulsions were developed using Final.)

emulsions *A* were exposed 6–12 months before the expiry date of the emulsion, the emulsions *B* 27 months before this date. The *B* emulsions were regarded as not having been stored. It is probably no chance occurrence that both *B* emulsion have much higher *CT* functions (Fig. 18). If IU emulsion *B* and ISS are compared, it is seen that they practically coincide. The reason can be an improved IU emulsion or a deterioration of the ISS emulsion due storage and aging, comparable with that obtained with emulsion of type *A*.

Since storage changes the emulsion continuously with time, it should be possible to treat the storing of the emulsion as a special component in the Fourier analysis. The problem could be treated as follows: If the transform of a fresh emulsion is denoted by $F_{em}(N; T)$ and that of a stored one by $F_{em}(N; T + t)$, then a Fourier transform will give the influence of the storage on the *A* emulsion, viz. $F_{em}(N; t)$. (*T*: date of manufacture, *t*: storage time.) If we assume that the original transform at time *T* is representative for the emulsion, then we obtain:

$$F_{em}(N; T + t) = F_{em}(N; t) \cdot F_{em}(N; T)$$

The $F_{em}(N; t)$ curve (Fig. 19) indicates the existence of two different phenomena. The minimum at low frequencies would indicate difference in the batches *A* and *B* while the decrease in $F_{em}(N; t)$ for the high frequencies would point to a storage phenomenon. It would seem to be natural that the older an emulsion becomes the more the limit of resolution and the contrast for the higher frequencies decrease. In the high frequency region the same phenomenon was observed with one batch of Kodak Microfile Pan. For the coarse grained and more sensitive IU emulsion on the other hand, it was not possible to register this high frequency tendency because of the increase in noise.

In intensity measurement, it is often necessary to know the contrast of an object. The question will be: How large contrast C_0 does an object need to have in order that a photographic emulsion will register a given contrast C_1 or in order

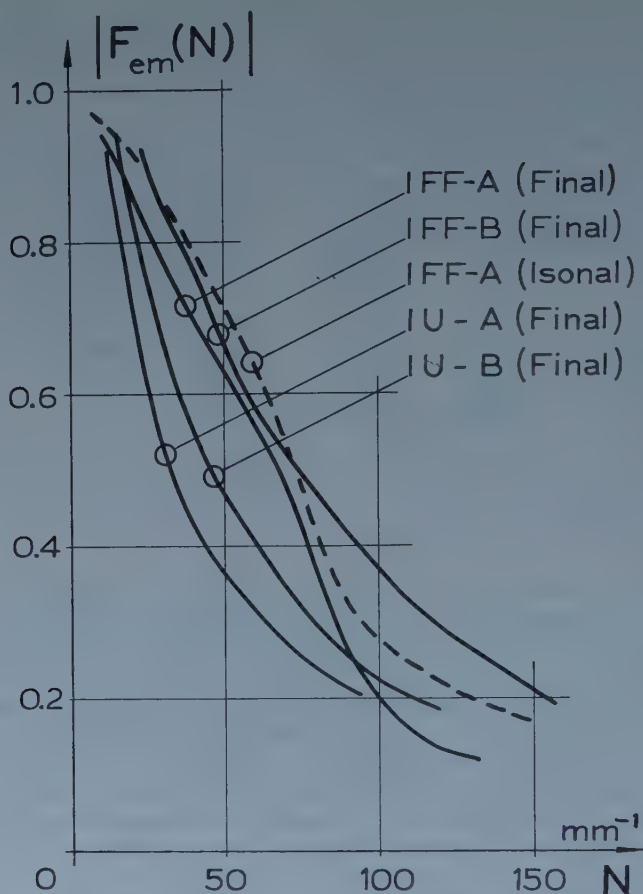


Fig. 18. $|F_{em}(N)|$ for two batches of IFF and IU respectively, developed with Final. IFF emulsion A in addition with Isonal.

that it will be registered at all? The CT functions given above provide only an indirect answer to this question. A different method for plotting the results would seem preferable in many cases (Fig. 20). One example is shown here in the form of curves for the B emulsion of IFF. In the figure, the solid curves represent the image of the sine pattern. The contrast C_o of the object is used as ordinate and the space frequency as abscissa. The parameters for the different curves are the contrasts of the images, viz. $C_i = 0.8, 0.6, 0.4$ and 0.2 . A detailed example will illustrate the idea behind the figure. If a sine pattern with a line density of 50 lines/mm is exposed, then the contrasts between 0.3 and 1.0 will be registered photographically as contrasts between 0.2 and 0.65. $C_o = 0.6$ will be registered as $C_i = 0.4$. If C_o is lower, then the image contrast may fall perhaps within the lower, shadowed region. The emulsion will then at least register the lines patchwise. It is therefore not possible to draw any sharp boundaries since so many factors exert an influence, e.g. the quality of the emulsion, storage,

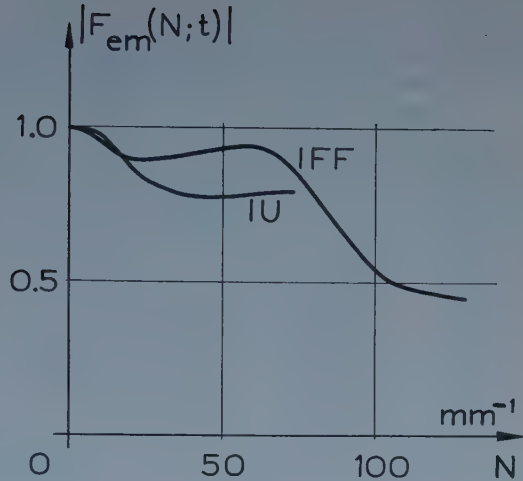


Fig. 19. The differences between two emulsion batches can be obtained by dividing the transforms of the two batches as $|F_{em}(N; t)|$. The storage time is in these cases > 14 months. The IFF curve indicates two different phenomena.

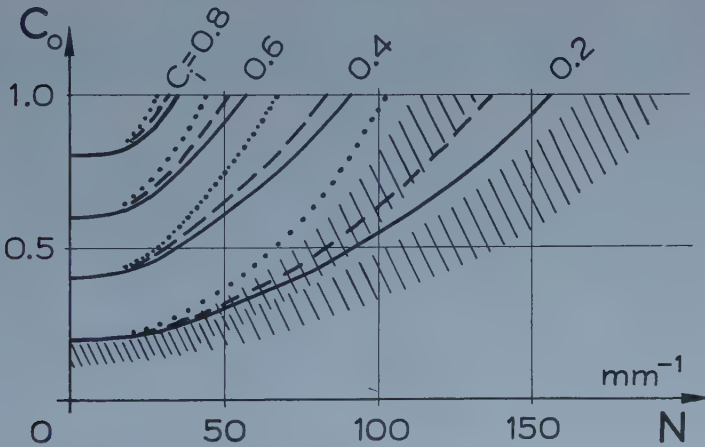


Fig. 20. These curves indicate how large a contrast an object must have in order to produce a given contrast in the image. For example, at $N=100 \text{ mm}^{-1}$, the emulsion $CT=0.2$ reproduces a sine-patterned object with a contrast of 0.55 if no lens system is used. The object must have $CT=0.71$ for reproduction with an aberration-free lens and $CT=0.95$ with the experimental optical system. The example refers to the IFF emulsion B. The diffuse regions indicate that the emulsion can resolve the line pattern more or less well depending on the quality of the emulsion and the development etc. The more bent region applies for reproduction with the optical system included, the lower one with only the emulsion.

development method etc. Object contrasts which are still smaller will not be registered. (This result has been obtained at an average density.)

If the object is however registered by means of an optical system, then the conditions will be less favourable because of the broadening effect of the lens system. If the CT function of the optical system is like the registration in Fig. 4, then the group of curves will bend more rapidly (dotted lines in Fig. 20). The shadowed region will also move upwards. The dashed curves give the conditions for a calculated Airy lens.

Table 5. Boundary frequencies, $N_{\max}$, of the image when the object was photographed using only the photographic emulsion. Only definite contrasts have been registered (in %).

Emulsion	Developer	$C_1 = 1.0$	$C_1 = 0.8$	$C_1 = 0.6$	$C_1 = 0.4$	$C_1 = 0.2$
		$N_{\max} \text{ mm}^{-1}$	$N_{\max} \text{ mm}^{-1}$	$N_{\max} \text{ mm}^{-1}$	$N_{\max} \text{ mm}^{-1}$	$N_{\max} \text{ mm}^{-1}$
IFF	Final	0	28-35 (A) (B)	53-57 (A) (B)	77-91 (A) (B)	100-156 (A) (B)
IF	Final	0	25	43	68	101
ISS	Final	0	19	32	58	108
IU	Final	0	17-22 (A) (B)	26-35 (A) (B)	44-60 (A) (B)	— -111 (A) (B)
IFF	Isonal	0	38 (A)	64 (A)	82 (A)	129 (A)

Table 6. Boundary frequencies, $N_{\max}$, of the image when the object was photographed using both the optical system and the emulsion. Only definite contrasts have been registered (in %).

Emulsion	Developer	$C_1 = 1.0$	$C_1 = 0.8$	$C_1 = 0.6$	$C_1 = 0.4$	$C_1 = 0.2$
		$N_{\max} \text{ mm}^{-1}$	$N_{\max} \text{ mm}^{-1}$	$N_{\max} \text{ mm}^{-1}$	$N_{\max} \text{ mm}^{-1}$	$N_{\max} \text{ mm}^{-1}$
IFF	Final	0	21-28 (A) (B)	40-45 (A) (B)	63-67 (A) (B)	87-103 (A) (B)
IF	Final	0	21	35	55	82
ISS	Final	0	17	28	46	80
IU	Final	0	16-20 (A) (B)	23-30 (A) (B)	38-48 (A) (B)	— -80 (A) (B)
IFF	Isonal	0	27 (A)	48 (A)	69 (A)	91 (A)

In both of the tables (Tables 5 and 6), it is shown how various coarse grained, high sensitivity emulsions effect the limiting values, $N_{\max}$. By $N_{\max}$, we mean the largest space frequency which can be registered with a predetermined contrast C_1 when the object has the contrast $C_0 = 1.0$ (Fig. 20). The values in the first of the tables correspond to the highest frequencies which the solid curves reach when only the emulsion is used for the registration. The second table gives the corresponding values for the dotted curves when a lens is added. Two emulsion batches, *A* and *B*, of the emulsions IFF and IU show that the limiting values are easily affected by uncontrollable factors. These values give an indication of the region of variation for $N_{\max}$. However, from the tables, it appears that $N_{\max}$ decreases as the emulsions become more sensitive. Exceptions exist, e.g. the IU emulsion *B* sometimes gives a higher value than ISS. One can also see that the IU emulsion *A* cannot register contrasts = 0.2 even if the object contrast = 1.0.

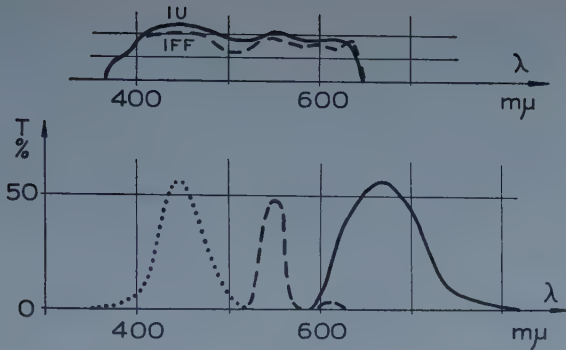


Fig. 21. Transmission curves for the three filters used. Wedge spectrogram for IFF and IU.

Influence of the developer

It is well known that a photographic emulsion is affected differently by different developers. In order to obtain information concerning the behaviour of the CT function, the IFF emulsion *A* was developed with Final and Isonal. Final is a fine grain, low contrast developer. It is known that Isonal works as a surface developer—the light distribution function is therefore decreased and the film speed increased. Isonal moreover tends to produce the adjacency effect. The difference between the CT functions also appears quite clearly. The emulsion developed in Isonal gave a higher CT function within the whole of the space frequency region (Fig. 18). The increase is approximately of the same order as that obtained upon changing from one film emulsion to another with the next lower sensitivity and graininess. This result is comparable to the conclusions from the previous section. The grain is affected differently by the developers. The mean grain size has decreased somewhat, as is confirmed by measurements of the Callier ratio [3]. The advantage which is obtained with a fresh emulsion, IFF-*B*, has been partly cancelled by the use of a different developer. The Isonal-developed emulsion has however not reached or passed the function values of the fresh *B* emulsion for $N > 75 \text{ mm}^{-1}$.

A subsequent article "The contrast transfer of periodical structures in a photographic emulsion developed with adjacency effects" will be published in this journal.

Different spectral regions

It was considered necessary to gain a more detailed knowledge concerning the influence of the different spectral regions on the shape of the light distribution for a black and white negative material. The interest has therefore primarily been to try and trace any distinct differences.

There are however many weaknesses in investigations of this type. The emulsions have a varying spectral sensitivity in the visible colour scale. In addition, the filters have different maximum transmissions and half value widths, etc. (Fig. 21). The blue filter is an interference filter with $\lambda_{\text{max}} = 456 \text{ m}\mu$. The green, coloured-glass filter with $\lambda_{\text{max}} = 546 \text{ m}\mu$ possesses the narrowest band. The red spectral band is far too spread out in the long wave part at the edge of, and

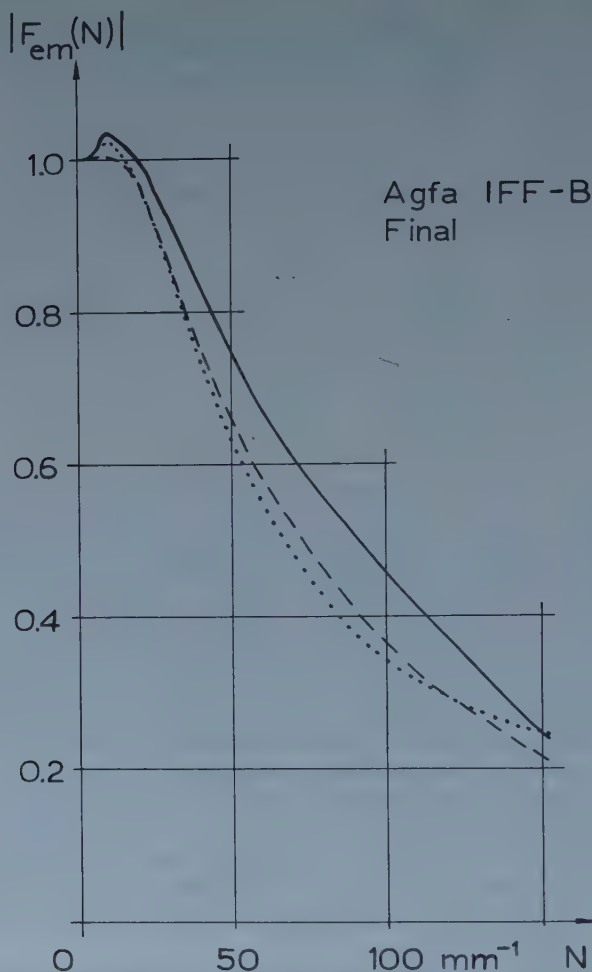


Fig. 22. Wave length dependence of IFF emulsion B. ····· CT curve for short wave length light (blue); ---- CT curve for medium wave length light (green); ——— CT curve for long wave length light (red).

also outside, the region of sensitivity of the emulsions. However, it has appeared that the relatively broad wave length region of the colours has not affected the clarity of the results. The lens system has been tested for all three spectral bands so that the usual Fourier calculation can be used purely experimentally and mathematically.

The two B emulsions of IFF an IU were investigated for all three colours. A series of measurements were performed in which every second cm of the film was exposed using blue, green and red light in that order. The contrast C_0 of the sine pattern was always 0.8. This series of three measurements was repeated five times. Thus each CT function corresponds to an arithmetical mean of five functions. Two density strips were exposed with each filter on the same piece

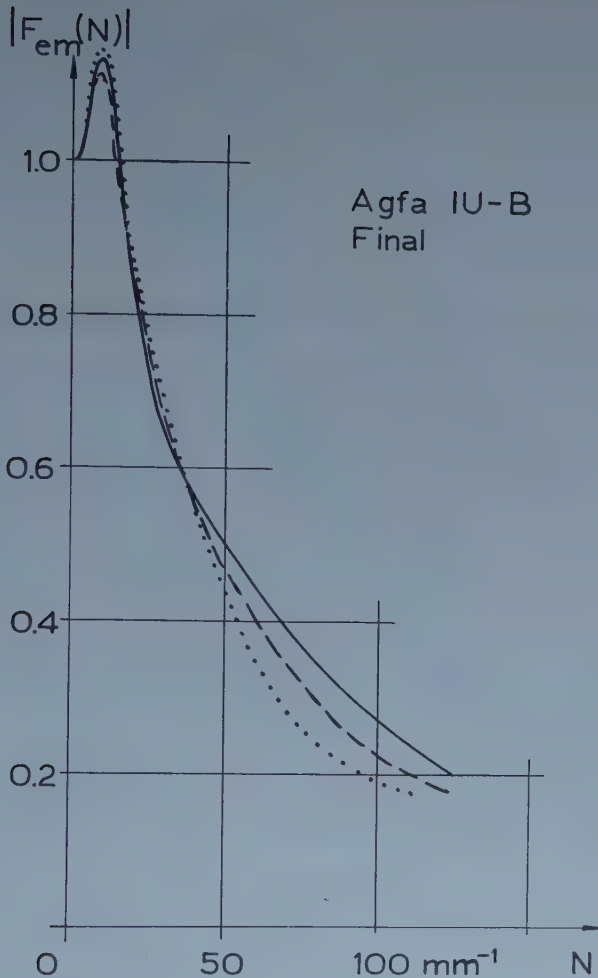


Fig. 23. $|F_{em}(N)|$ is dependent on the colour composition of the exposure. IU emulsion B. $|F_{em}(N)|$ for short wave length light (blue); ---- $|F_{em}(N)|$ for medium wave length light (green); — $|F_{em}(N)|$ for long wave length light (red).

of film. Thus, the light diffusion functions of the different colours were recorded using one single piece of film and an unvariable development procedure. A comparison of the light distribution of the different colours (Fig. 22 and 23) shows that they are quite separate within the greater part of the measurable space frequency region. Now, it is known that the light distribution is dependent upon, amongst other things, the spreading of the light at each individual grain in the photographic emulsion and also on the absorption of light in the emulsion layer. It is also known that blue light causes the largest spreading. In return, the emulsion absorbs the short wave light more strongly—and this light therefore has a shorter path in the emulsion. These effects when combined together have a minimum for green light. The curves obtained show however that, when the effects

are combined together, the spreading dominates over the absorption. For all space frequencies N with IFF and for $N > 40 \text{ mm}^{-1}$ with IU it was found that the curves gave the highest CT values with red light. With the coarse grained emulsion, the CT curves of the three colours coincided in the low frequency band. Both of the blue curves lie in general lowest. This agrees with the assumption that the blue light is spread out the most.

The differences in the CT functions do not depend on the different grain sizes. Measurements of the Callier ratio gave namely the same mean grain size for an emulsion independent of the wave length region of the exposures.

Transformation of $|F_{\text{em}}(N)|$ to the light distribution function $\phi_{\text{em}}(x)$

Several methods can be considered for the Fourier transformation of the $|F_{\text{em}}(N)|$ curves in order to obtain the physically interesting $\phi_{\text{em}}(x)$. A purely analytical inversion would be desirable but, as will be shown, such a procedure is not practical because of the complexity of the functions. A comprehensive manual calculation of $\phi_{\text{em}}(x)$ would be too time-consuming. A much quicker and more accurate inversion can be performed by means of an electronic calculating machine. The $\phi_{\text{em}}(x)$ curve is then obtained in the form of code values which can however be easily interpreted. The accuracy is then mainly determined by the accuracy of the experimental curve and by the interval division employed for the calculations with the machine.

Certain theoretical assumptions involving one-parameter functions were made in order to provide a comparison between the different emulsions. The following conditions must be taken into consideration throughout:

- (1) $F_{\text{em}}(0) = 1$ (normalization)
- (2) $F_{\text{em}}(N_{1/2}) = \frac{1}{2}$. (Motivated by the fact that an interpolation within the middle frequency region is considered to give function values with the greatest accuracy. The frequency $N_{1/2}$ for the different emulsions is given in Table 7.)

Table 7.

Emulsion	Developer	$N_{1/2}$ (mm^{-1})	N_{max} (mm^{-1})	$^{1/2} \cdot \phi_{\text{em}}(0)$ (μ)	$^{1/10} \cdot \phi_{\text{em}}(0)$ (μ)
IFF - A	Final	66	116	4.8	12
IFF - B	Final	73	145	3.2	10
IF	Final	57	116	4.2	13
ISS	Final	43.5	116	4.0	16
IU - A	Final	33.5	72	—	—
IU - B	Final	46	116	3.9	16
IFF - A	Isonal	73.5	145	3.6	11

- (3) $|F_{\text{em}}(N)|$ is experimentally determined within a limited region.

Four assumptions were considered in the work. Both the Fourier transforms and the light distribution functions are given. C_0 and σ are equal to constants. The assumptions were as follows:

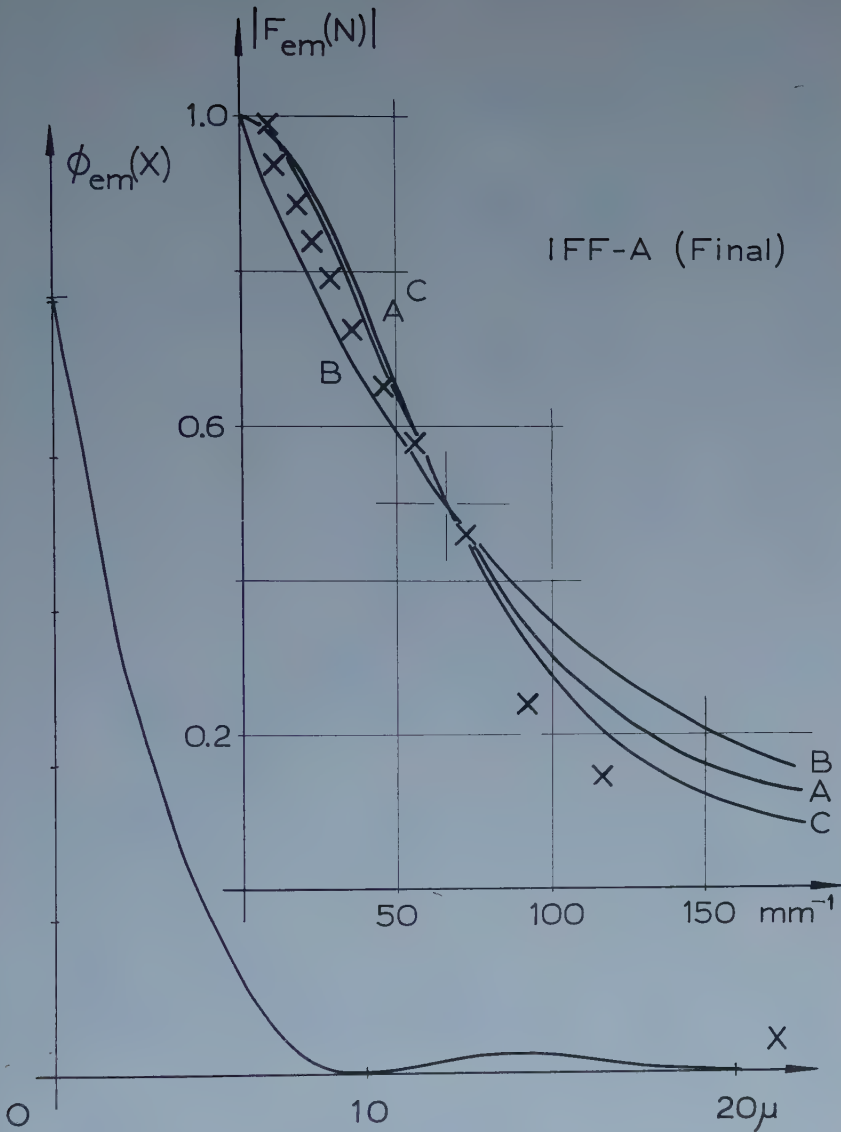


Fig. 24.

Fig. 24-30. Fourier transform $|F_{em}(N)|$ on the right. The transformed light distribution functions $\phi_{em}(x)$ on the left. $\times$ indicates experimentally determined points; A, assumption according to Frieser; B, assumption which is "inverse" to the previous one; C, assumption according to Sayanagi; D, assumption according to the Gauss function.

(A) Assumption by Frieser [4],

$$F(N) = \left[1 + \left(\frac{N}{N_{1/2}} \right)^2 \right]^{-1}; \quad \phi(x) = C \cdot e^{-2\pi N_{1/2} \cdot |x|}$$

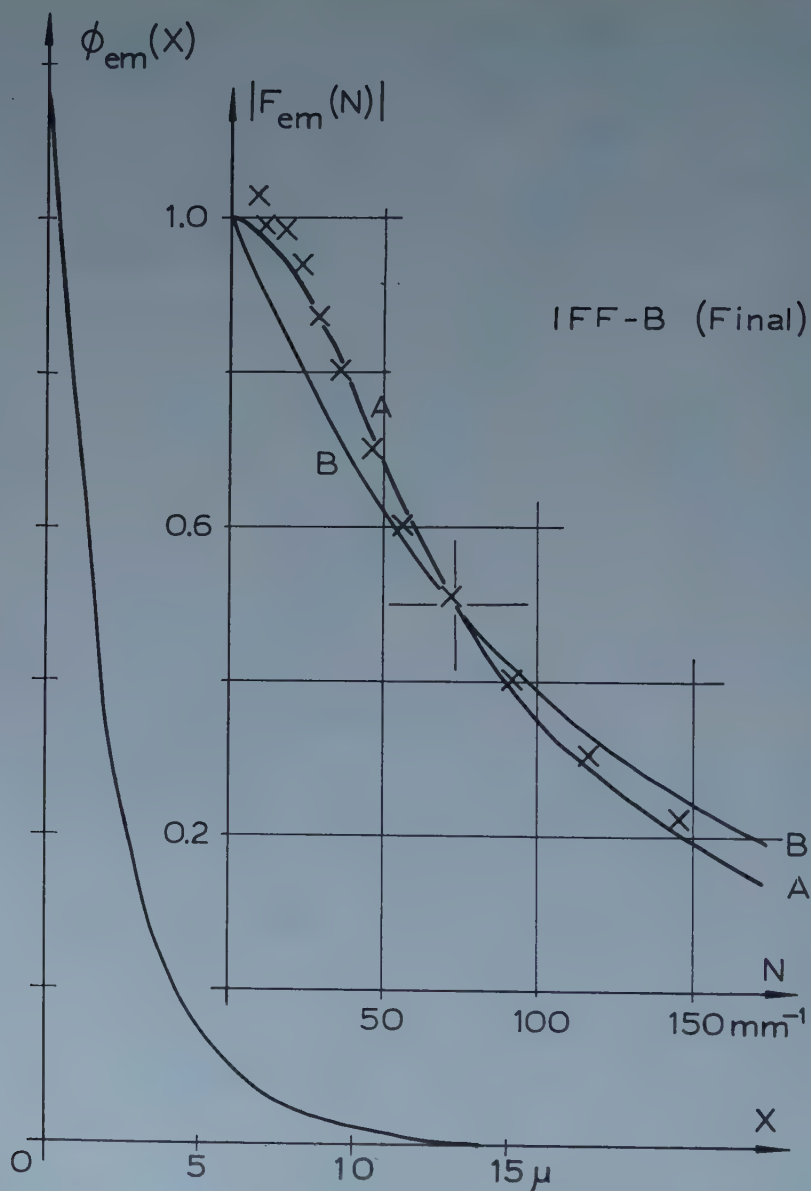


Fig. 25.

(B) An assumption which is also conceivable, at least if small N values are excluded, is

$$F(N) = 2^{-\frac{N}{N_{1/2}}}; \quad \phi(x) = C \cdot \left[1 + \left(\frac{2\pi N_{1/2}}{\ln 2} \cdot x \right)^2 \right]^{-1}$$

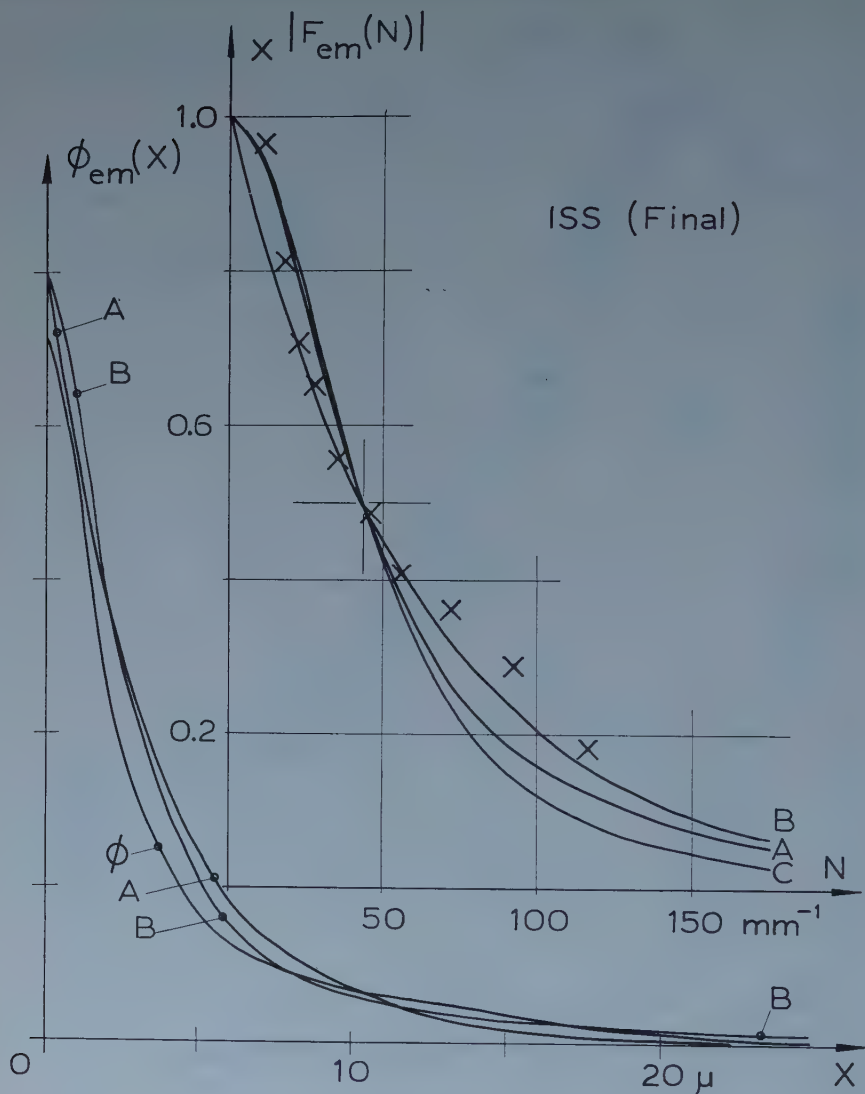


Fig. 27.

The $\phi_{\text{em}}(x)$ curves were Fourier transformed from $|F_{\text{em}}(N)|$ by means of an electronic calculating machine, BESK. The interpolation interval $\Delta N = 10 \text{ mm}^{-1}$ was sufficiently narrow since the primary data for the CT functions were measured using a corresponding interval up to $N \sim 116 \text{ mm}^{-1}$ (Table 7). At the measuring boundary, $|F_{\text{em}}(N)| \sim 0.2$. The extrapolation curve for $N > 116 \text{ mm}^{-1}$ had to be drawn with a slight bend which fitted smoothly on to the measured curve. The boundary for N , where $F_{\text{em}}(N) = 0$, was considered to lie at some point between 350 and 500 mm^{-1} which was different for different functions. These relatively high values were supported by the fact that exposures with the frequency

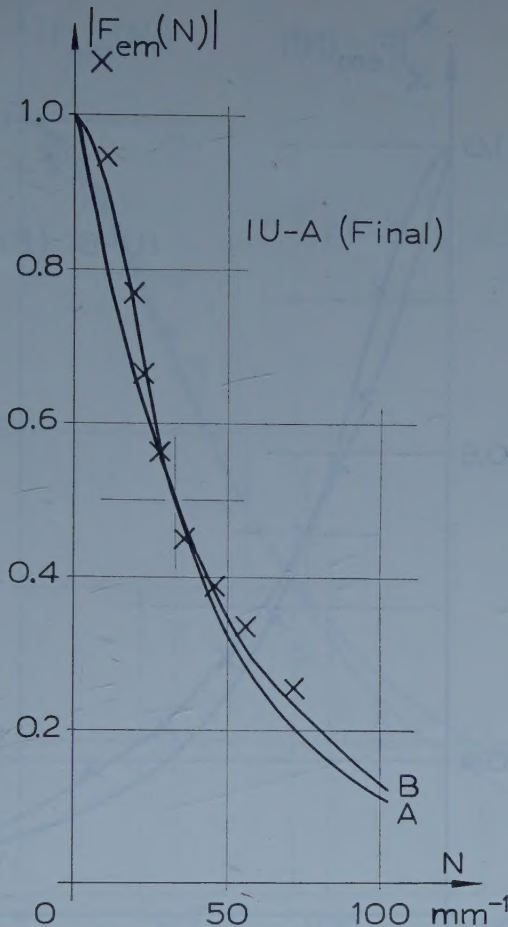


Fig. 28.

$N = 240 \text{ mm}^{-1}$ produced a pattern which appeared sporadically amongst the noise of the emulsion. A harmonic analysis was not considered necessary. On the other hand, the unknown parts of the curve were investigated by means of extrapolation using definite functions or a free-hand drawing of the curve. The functions were

$$F(N) = 0, \quad F(N) = e^{-\frac{N}{C}}, \quad F(N) = C_1 \cdot e^{\frac{N}{C_2}} \quad \text{and} \quad F(N) = \frac{C_1}{C_2 + N^2}.$$

The first two extrapolation functions, and in particular the first, gave a rapidly oscillating function $\phi(x)$ (Gibbs phenomenon) due to the sharp bends in the curves. With the other extrapolation procedures, one of the constants was determined from the condition that $\frac{dF(N=116)}{dN}$ is the same for the experimental and the extrapolated curves. The expression

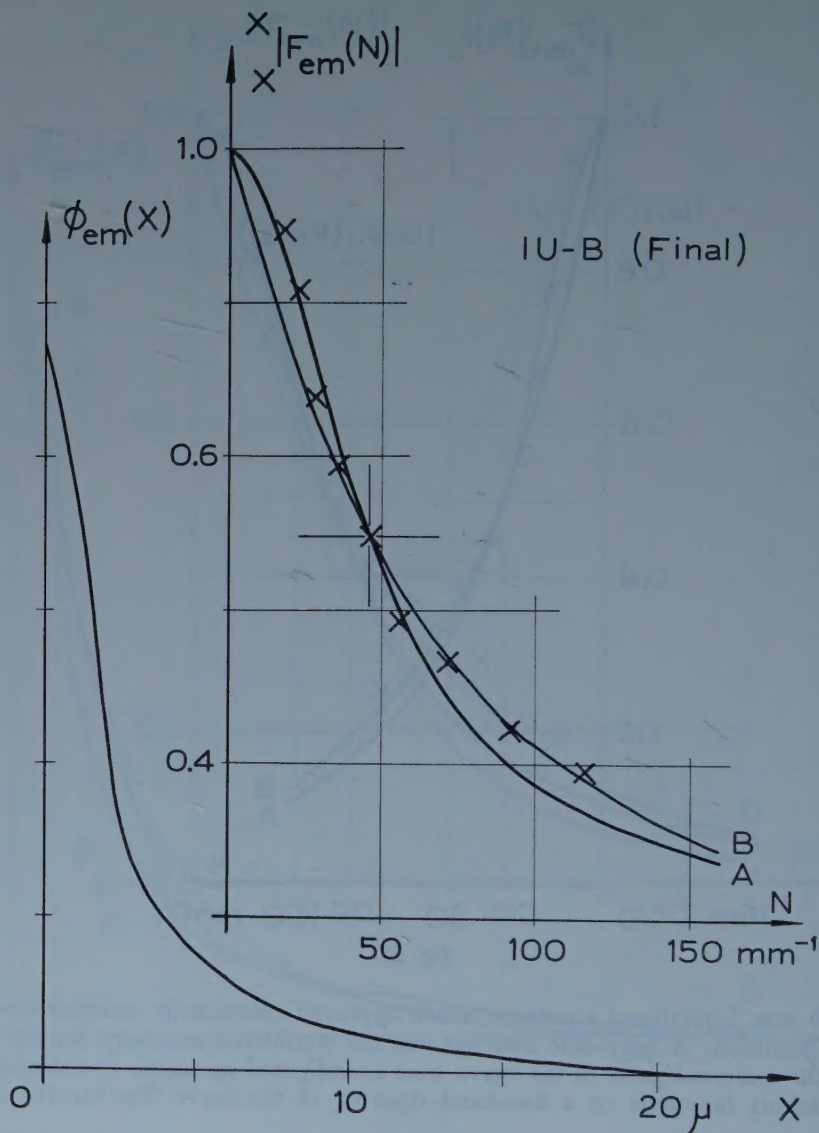


Fig. 29.

$$\phi_{em}(0) = \int_{-\infty}^{+\infty} F_{em}(N) dN$$

gave an estimated accuracy in $\phi_{em}(0)$ of at most $\pm 5\%$. A free-hand extrapolation could therefore be employed with advantage.

In Figs. 26 and 27, a more complete treatment is made for IF and ISS in order to demonstrate the relation of the different theoretical functions to each

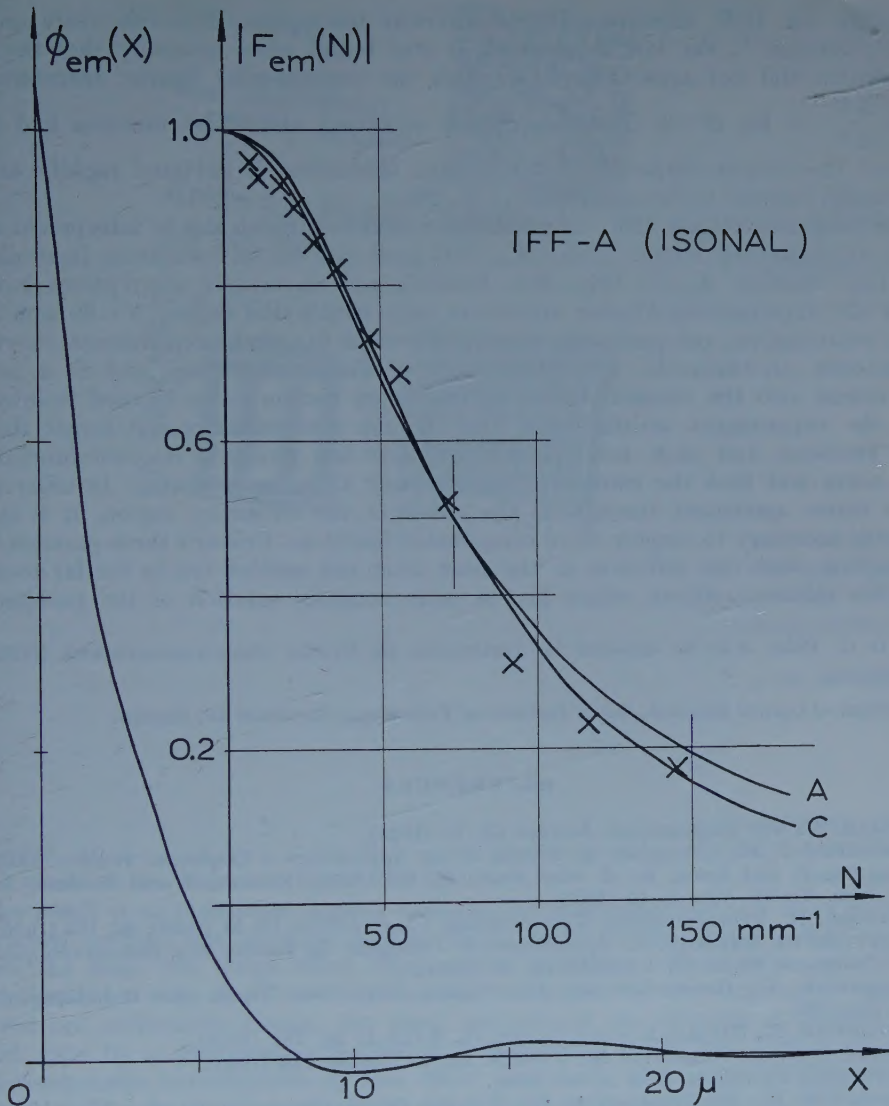


Fig. 30.

other and to the experimental curve. There are also assumptions compared to the light distribution function. The central part of $|F_{em}(N)|$ has, in fact, been accurately investigated. However, with $\phi_{em}(x)$, the region $|x| < 2\mu$ is determined to a considerable extent by the high frequencies of $|F_{em}(N)|$ which have here been extrapolated. The assumptions are indicated in the figures 24-30.

It was in general found that the assumptions of Frieser and Sayanagi fit much better than the other two. The emulsions which followed Frieser's assumption well were IFF-B and IF. Even Sayanagi's assumption was satisfactory here,

especially for IFF. However, $IU - B$ followed assumption B satisfactorily only for $N > 30 \text{ mm}^{-1}$. For low frequencies, it was found quite generally that the B assumption did not agree completely with the experimental Fourier transforms since $\frac{\Delta F}{\Delta N} \neq 0$ for $N = 0$. The assumption involving the Gauss function had in general the correct shape for $N < N_{1/2}$. But, thereafter, it deviated rapidly and eventually became quite unsuitable (Fig. 26).

The Isonal-developed IFF - A exhibited a tendency which can be interpreted as being an adjacency effect. Thus, $|F_{\text{em}}(N)|$ gave rise to an oscillating light distribution function $\phi_{\text{em}}(x)$ (Fig. 30). Frieser's and Sayanagi's assumptions both fitted the experimental Fourier transform only within the region $N < 80 \text{ mm}^{-1}$.

An unambiguous, one-parameter assumption which fits all the experimental curves is probably unobtainable. The assumptions of Frieser, Sayanagi, and to a certain extent also the classical Gauss curve can in certain cases be used to interpret the experiments satisfactorily. One should also certainly not forget that each emulsion and each development procedure has probably its own function as a norm and that the emulsion function may alter upon storing. In order to get a better agreement throughout the whole of the frequency region, it is apparently necessary to employ more complicated functions. Frieser's three-parameter assumption with one diffusion in the near zone and another one in the far zone, i.e. two different effects, might give a more accurate solution of the problem.

P. O. G. Palm is to be thanked for performing the Fourier transformations with BESK (Stockholm).

Institute of Optical Research, Royal Institute of Technology, Stockholm 70, Sweden.

REFERENCES

1. CLARK, W., *The Photographic Journal* 65, 76 (1925).
2. DUFFIEUX, P. M., *L'Intégrale de Fourier et ses Applications à l'Optique*. Rennes, 1946.
3. EGGERT, J. and KOCH, B., *Z. wiss. Phot.* 49, 104 (1954); EGGERT, J. and KÜSTER, A., *Wiss. Veröff. Agfa* 5, 49 (1935).
4. FRIESER, H., *Photographische Korrespondenz* 91, 69 (1955); 92, 51 (1956); 92, 183 (1956).
5. GAULTIER DU MARACHE, J., *Applications de l'intégrale de Fourier aux photographiques*. *Cahiers de Phys.* 39, 1 (1952); 40, 29 (1952).
6. INGELSTAM, E., *Berichtsheft der Jena-Tagung Nov. 1958 "Optik aller Wellenlängen"*, page 286.
7. INGELSTAM, E., DJURLE, E. and SJÖGREN, B., *J.O.S.A.* 46, 707 (1956).
8. JONES, L. A. and NELSON, C. N., *J.O.S.A.* 30, 93 Comm. 748 (1940).
9. LINDBERG, P., *Optica Acta* 1, 80 (1954).
10. MEES, C. E. K., *The Theory of the Photographic Process* Chapter 6, 199, New York. 1954.
11. RONCHI, V., *Z. Wiss. Phot.* 39, 2 (1940).
12. SAYANAGI, K., *J.O.S.A.* 47, 566 (1957).
13. SELWYN, E. H. W., *The Photographic Journal* 88 B, 6 (1948).

Tryckt den 17 februari 1960

Uppsala 1960. Almqvist & Wiksells Boktryckeri AB